

行政院國家科學委員會專題研究計畫 成果報告

供應鏈中退化性產品在儲存空間有限且允許延遲付款下的 最佳存貨訂購策略之研究(I) 研究成果報告(精簡版)

計畫類別：個別型
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執行期間：96年08月01日至97年07月31日
執行單位：致理技術學院企業管理系(科)

計畫主持人：廖瑞容

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供應鏈中退化性產品在儲存空間有限且允許延遲付款下的最佳存貨訂購策

略之研究(I)

Optimal lot size of the inventory models for deteriorating items with capacity
constraint and trade credit financing in a supply chain system (I)

計畫編號：NSC 96-2221-E-263-003-

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主持人：廖瑞容 致理技術學院企業管理系

1. Scope and Purposes

As we all know, the supplier will offer the retailer a trade credit period to induce a huge orders, an important problem associated with inventory maintenance is to decide where to stock the items. Maybe rented warehouse is used when the ordering quantity exceeds the limited capacity of the owned warehouse.

Therefore, we study a deterministic inventory model for deteriorating items under limited storage capacity (own warehouse and rented warehouse) and permissible delay in payments is developed to help the decision-maker to make the decision. Herein, it is assumed that deterioration rates of items in the

two warehouses may be different. Firstly, expressions are derived for the total a cost of the inventory system, respectively. Secondly, for these models, the optimal solution not only exists but also is unique. Thirdly, a theorem is developed to determine the optimal cycle time and order size. Finally, sensitivity analysis of the optimal solution with respect to the parameters of the system is carried out.

2. Literature Review

Recently, several researchers have discussed a two-warehouse inventory system such as Hartely (1976). In this system, it is assumed that the holding

cost in RW is greater than that in OW. Hence, the items are stored first in OW and only excess stock is stored in the RW. Sarma (1983) developed an inventory model with infinite replenishment rate and two levels of storage. In the above models, the analysis is carried out without taking shortages. Goswami and Chaudhuri (1992) further developed two-storage models with and without shortages, allowing time -dependent demand. Bhunia and Maiti (1998) developed a two warehouse model for deteriorating items with linearly trend in demand and shortages.

Additionally, several authors assume the deterioration in both warehouses such as Sarma (1987) first developed a two-warehouse model for deteriorating items with the infinite replenishment rate and shortages. Pakkala and Achary (1992a, 1992b) further considered the two-warehouse model for deteriorating items with finite replenishment rate and shortages. Hiroaki and Nose (1996) discussed an inventory perishable inventory control with two types of

customers and different selling prices under the warehouse capacity constraint. Benkherout (1997) relaxed the assumption of Sarma (1987) to be stocked in the OW. Zhou (1998) presented a two-warehouse model for deteriorating items with time-varying demand and shortages during the finite planning horizon. Yang (2004) developed the two-warehouse model for deteriorating items with shortages and inflation. Zhou and Yang (2005) established a two-warehouse model with stock-level-dependent demand rate and taking the transportation cost into account.

Traditionally, the effect of a permissible delay in payments on the optimal inventory system has received the attention from many researchers. Goyal (1985) derived an EOQ model under the conditions of permissible delay in payments. Thereafter, some research articles related to Goyal such as Chand and Ward (1987) analyzed Goyal's problem under assumptions of the classical economic order quantity model.

Aggarwal and Jaggi (1995) considered an EOQ model with an exponential deterioration rate under the condition of permissible delay in payments. Chu et al. (1998) examined the economic ordering policy of deteriorating items in Aggarwal and Jaggi's model. Jamal et al. (1997) further generalized Aggarwal and Jaggi's model to allow for shortages. Shah (1993a) extended an EOQ model in which delays in payment were permissible and items in inventory deteriorated at a constant rate over time. Chen and Chung (1999) analyzed buyer's economic order model under trade credit. Davis and Gaither (1985) developed an EOQ model for firms offering a one-time opportunity to delay payments for the order of a commodity. Shah (1993b) developed a probabilistic time-scheduling model for an exponentially decaying inventory when payment delays were permissible. Hwang and Shinn (1997) developed a retailer's pricing and lot-sizing policy for exponentially deteriorating products under the condition of permissible delay in payments. Sarker et al. (2001)

developed an optimal payment time under permissible delay for payment for products with deterioration. The other relevant papers related to the delay of payments such as Arcelus and Srinivasan (1993), Chung (1998), Chung et al. (2001), Chung and Liao (2004, 2006), Daellenbach (1986), Haley and Higgins (1973), Jaggi and Aggarwal (1994), Jammal et al. (2000), Shinn (1997), Shinn and Hwang (2003) and their references.

3. Notations and Assumptions

Notation

D =the demand rate per unit time ;

A =the replenishment cost per order ;

W =the storage capacity of the owned warehouse ;

T =the length of replenishment cycle ;

Q =the replenishment quantity per replenishment ;

s =the selling price per unit item ;

c =the purchasing cost per unit item ;

h_0 =the holding cost per unit per unit time in OW ;

h_r =the holding cost per unit per unit time in RW ;

α = the deterioration rate in OW ;

β = the deterioration rate in RW ;

$I_0(t)$ = the inventory level in OW at time t ;

$I_R(t)$ = the inventory level in RW at time t ;

T_w = the time at which the inventory level reaches zero in RW ;

$$T_a = \frac{1}{\alpha} \ln\left(1 + \frac{\alpha W}{D}\right) ;$$

M = the trade credit period ; ($M > T_a$)

I_p = interest charges per \$ investment in inventory per year ;

I_e = interest which can be earned per \$ per year ;

Assumption

- (1) Replenishment rate is infinite, and lead time is zero.
- (2) The time horizon of the inventory system is infinite.
- (3) Shortages are not allowed to occur.
- (4) The owned warehouse (OW) has a fixed capacity of W units.
- (5) The rented warehouse (RW) has unlimited capacity.
- (6) The items of OW are consumed only after consuming the items kept in

RW.

(7) The inventory costs (including holding cost and deterioration cost) in the RW are higher than those in the OW. That is, $h_r + \beta c > h_0 + \alpha c$.

(8) The sales revenue generated during the credit period is deposited in an interest bearing account with rate I_e . At the end of this period, the account is settled and the remaining balance will be paid off by loan from the bank with interest charges in stock with rate I_p if necessary. After the credit period, the retailer makes payment to the bank immediately after the sales of the items until the retailer pays off the remaining balance.

4. Conclusions

The main purpose of this paper is threefold: Theorem 1 shows that the optimal solution not only but also is unique. Theorem 2 presents an optimal solution procedure to find the ordering policy. Finally, numerical examples are solved and the sensitivity of the solution to changes in the values of different

parameters has been discussed.

5. Self-Evaluation

This research corresponds to the original plan and has attained its aim. Hence, the study is of great academic value and suitable for publication in academic journals.

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