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計 畫 主 持 人 ：張正昌

計畫參與人員：此計畫無其他參與人員：陳珠龍
碩士班研究生-兼任助理人員：江貴明
碩士班研究生-兼任助理人員：謝玉瑜
大專生-兼任助理人員：黃姿華
大專生-兼任助理人員：林奕君

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PA-DSAS 推進策略下多區域投資資金配置模型初探

摘　要

多區域多營運據點海外直接投資已被視爲是服務產業獲得長期競爭優勢之中要手段，亦被視爲是阻止潛在競爭者搶奪市場之障礙。然而，目前之研究文獻尚無針對服務產業之資本預算限制下海外市場拓模型。基此，本文旨在應用專案推移(project advancement, PA)理論中之分散化同步推進策略(DSAS)，提供一含具有多區域多營運據點之海外市場拓模型，以求取資本預算限制下之最適商品訂價與資本投資政策。於發現一些模型結構性質後，本文應用分段線性近似、線性分式規劃與加權法提供一有效求解方法。本文之結果可應用於速食業、牛排館與咖啡廳等服務產業。

關鍵詞：服務產業、多區域多營運據點、市場拓、分段線性、分式規劃

Modeling The Multi-site Locations Distribution of Capital Funds on Adopting DSAS Advancement Strategy of PA

Abstract

The foreign direct investment of multi-site locations has been identified as one of the most important measures for gaining long-term competitive advantages, and has been argued to serve as barrier to the entry of potential competitors in service industries. However, the capital budget constrained foreign expansion models may have far reaching the implications for the service sectors. The paper aims to present a foreign expansion model with multi-site locations based on the concurrently investment framework, called the decentralized synchronous advancement strategy (DSAS) defined by the theory of project advancement (PA). The proposed model is a special case of maximum buffer time model, which is capable of optimizing the commodity pricing and capital investment policy. Having found some properties of the model, we propose a solution method which consists of the techniques of piece-wise linear approximation, linear fractional programming and weighting method. The results of proposed model may be beneficial for foreign expansion in services with regard to business like fast food, steak restaurant, café shops, and so on.

Keywords: Service industry**,** multi-site locations, markets expansion, piece-wise linear, fractional programming

1. Introduction

Service firms usually offer large percentage of intangible outputs which have long been acknowledged to differ from purely physical outputs (Dunning, 1989; Hirsch, 1993). Today, the value of services exceeds the value of manufactured, tangible outputs. Service accounts for more than half of the gross domestic product in all developed countries and in most developing economies (Clark and Rajaratnam, 1999). As trade negotiation continues to lower barriers to services, more markets are open to services (King, 2003). Because service firms usually provide the outputs with the characteristic of inseparability (Brouthers et al., 2002), directly making an investment in a host country is the main means for delivering a service abroad - generally referred to as foreign direct investment (FDI). Thus, market-seeking (or market-developing) is usually the principal motive of a service provider for FDI in some countries.

The FDI's growth success of a service firm is closely related to the decision-making well in entry mode, locations, capital budgeting, and agency scheduling. It is due to the reasons such as more specialized professional skills, knowledge, and customization; service providers have been shown they usually prefer high control entry modes in highly competitive markets (Erramilli and Rao, 1993; Brouthers et al., 2002; Bouqet et al., 2004). Thus, wholly-owned based FDI is the main approach for seeking product market abroad in service industries.

Of driving a wholly-owned based market-seeking FDI, the selections of country-based locations are the basic concern of a service provider. A number of determinants influence on the choices of country-based locations of a service provider. Most empirical studies related to FDI determinants explained why firms in the manufacturing sector prefer to locate in some countries and not in others. Market size, internationalization of host country, and index of host country business environment have been identified as the most important determinants to a manufacturer's FDI decision-making (Dunning and Norman, 1987; Culem, 1988; Pearce, 1991). Studies on the determinants of foreign expansion in services are far fewer, but most scholars agree that FDI determinants of manufacturing can be applied to services (Seymour,1987; Nigh et al., 1986; Goldberg and Johnson, 1990). Also, Kundu and Contractor (1999) argued that sector-specific factor is also a critical determinant for services except the above determinants.

Having decided the country-based locations for driving a wholly-owned based market-seeking FDI, a service provider usually has further to give priority to a number of sites so that its service-agencies are capable of in close proximity to the customers living in those sites as a consequence of limited set of resources, such as capital budget, professional manpower, international management experiences, etc. Many scholars believe, of a specific product market, a powerful and loyal customer base may serve as a barrier to entry (Dan, 1978; Li, 1994; Cloninger, 2004). In particular, the intangibility of services creates difficulties for service firms because potential customers have trouble identifying differences in services offering (Mitchell and Greatorex, 1993; Campbell and Verbeke, 1994). Indeed, it is reasonable that suppose a powerful and loyal customer base would provide a service firm with an extremely good reputation to be spilled over whole target market. Further, the product/services reputation has been shown as an influence on consumer's perceived quality, and perceived value, which lead to purchase and repurchase intentions (Dodds et al., 1991; Zeithaml, 1988; Chang and Wildt, 1994; Jayant and Ghosh, 1996; Petrick, 1999; Woodruff, 1997). Clearly, this preferential spillover effect may lead to an increase in perceived risk for a potential competitor while attempting to enter that said market.

In spite of the role of this spill-over effect has been relatively neglect until relatively recently, it would highlight much of importance of the budgeting and scheduling modeling in regards to foreign expansion of multi-site locations. Indeed, toward achieving multi-site locations, the consumers in each location even in each detailed site have a substantial strategic-value; one usually needs to further find an appropriate budgeting and scheduling policy, thereby realizing the desires of a service provider as possible. It is only recently however, that academics have paid attention to this FDI-modeling and, in particular, focused on manufacturing, and not services. Efficiency-seeking, in general, is the principal motive of a manufacturer for FDI in multi-site locations. Indeed, most of labor-intensive manufacturing sectors motivate on finding a product supply market that has comparative advantage such as abundant raw materials, low cost labor and specific skill. International facility location problem (IFLP) is the main model related to efficiency-seeking FDI and examines the issues such as facility locations choices, open periods, productive capacity design, and production distribution, and so on. (Rosing,1994; Veter and Dincer,1995; Myung et al., 1997; Hinojosa et al., 2000; Carrizosa and Conde, 2002; Bhutta et al.,2003; Bhutta, 2004). Just as described in previous, service firms usually provide the outputs with the characteristic of inseparability, thus market-seeking is usually the principal motive of a service provider for FDI in multi-site locations and any IFLP may not be applied to services.

It is assumed here that the country-based locations have been decided for driving a wholly-owned based market-seeking FDI. Also, a number of detailed sites in each country have taking priority of making investment, thereby developing a loyal customer base to serve as a barrier to entry. Of critical importance in the context of this paper is the modeling of marketing-seeking FDI as stated above, particularly, the finding of an effective budgeting and scheduling policy so that the desires of a service provider are realized as possible. The paper will be organized as follows: a nonlinear programming approach is employed to formulate our concern in Section 2, some assumptions relating to the proposed model are given in Section 3 in order to obtain analytical results. Based on the analytical results, a solution method consisting of the techniques of piece-wise linear approximation, linear fractional programming and weighting method is then presented in Section 4. Finally, an illustrative example is given in Section 5.

2. The problem

We consider a foreign expansion program with multi-site locations in service industries. A service firm is assumed to consider the following case of wholly-owned based FDI. Toward seeking a specific N-country product market, there are L_j service-agencies that have taken priority of being opened in a selected city at time 0, where located in country j $(j = 1,2,...,N)$, in order to develop a loyal customer base to serve as a barrier to entry. Nevertheless, it is assumed here that the invested capital to each agency may be differentiated as two items. They are respectively "capital investment for delivery (CIFD)" and "capital investment for production (CIFP)." Also, each country $j = 1,2,...,N$ is assumed to be assigned a target cost basis of capital, $CA_j^{taget}(t)$ at time t as the following form.

$$CA_j^{taget}(t) = \begin{cases} CA_j^{taget}(0), & t < T_j \\ CA_j^{taget}(T_j), & t \geq T_j \end{cases} \tag{1}$$

Thus, if we let $CIFD_j^*(t)$ and $CIFP_j^*(t)$ represent the target CIFD and the target CIFP at time t, respectively, then we get

$$CA_j^{taget}(t) = \begin{cases} CIFP_j^*(0) + CIFD_j^*(0), & t < T_j \\ CIFP_j^*(T_j) + CIFD_j^*(T_j), & t \geq T_j \end{cases} \tag{2}$$

In order to achieve the goal—realizing $CA_j^{taget}(T_j)$ at time T_j, the service firm is assumed to consider the following type of finance policy. The target CIFD to the agencies planned to be opened at time 0 has been budgeted, but the CIFP has been established a design-to-budget goal, B_0. Instead, B_0 is understood to be a constraint on the total invested capital for production to all agencies planned to be opened at time 0. Thus, the total CIFP to all agencies planned to be opened at time 0 has to be below B_0. Indeed, if we let $c_j(0)$ denote the invested amount in CIFP at time 0 to each agency in country j, then we get

$$\sum_{j=1}^{N} L_j \cdot c_j(0) \le B_0 \qquad (3)$$

Further, in order to gain the capital budget enough to make an increase in capital at time T_j for each agency, it is assumed that the earnings gleaned by the L_j agencies (opened at time 0) are capitalized, thereby providing themselves with budget for capital increased at time T_j for each country $j = 1,2,...,N$. Considering there are infinite alternatives that are available relating to make investment in CIFP to each agency. Each alternative may be viewed as a specific production-design proposal. The larger investment in CIFP leads to the higher commodity quality. Indeed, notation $c_j(0)$ not only denotes the invested amount in CIFP at time 0 to each agency in country j but can viewed as a specific level of production-design, costing $c_j(0)$. Thus, if the service provider attempts to the increase commodity quality via promoting the level of production-design up to level $c_j^*(T_j)$ from current level $c_j(0)$, then the additional CIFP needs to be increased would exceed the amount of $c_j^*(T_j) - c_j^*(0)$ in general. Accordingly, we define the capital function relating to CIFP increased as follows:

$$C_j(c_j(0), c_j(T_j)) = \xi_j \cdot \{c_j^*(T_j) - c_j(0)\}, \qquad (4)$$

where $c_j^*(T_j)$ denotes the desired level of production-design to each agency at time T_j, C_j denotes the additional CIFP needed to be increased in order to promote the current level of production-design up to level $c_j^*(T_j)$, and ξ_j is a parameter not less than 1.

In addition, the target CIFD to each agency at time T_j is assumed here to be the same as the target CIFD at time 0. Accordingly, if we let $CA_j(0)$ be the actually invested capital at time 0 for country j, we see that

$$\begin{aligned} CA_j(0) &= CIFD_j^*(0) + L_j \cdot c_j(0) \\ &= L_j \cdot [\overline{CIFD_j^*}(0) + c_j(0)] \end{aligned} \tag{5}$$

$$\begin{aligned} CA_j^*(T_j) &= CIFD_j^*(T_j) + CIFP_j^*(T_j) \\ &= L_j \cdot \overline{CIFD_j^*}(0) + L_j \cdot c_j^*(T_j) \end{aligned} \tag{6}$$

where $\overline{CIFD_j^*}(0)$ denotes the target CIFD at any time point to each agency in country j and thus $CIFD_j^*(0) = L_j \cdot \overline{CIFD_j^*}(0)$.

With the use of the results of (4) - (6), it follows that

$$\begin{aligned} TNR_j^{target} &= CA_j^*(T_j) - CA_j^*(0) + (\xi_j - 1)\{c_j^*(T_j) - c_j(0)\} \\ &= L_j \zeta_j \{c_j^*(T_j) - c_j(0)\}, \end{aligned} \tag{7}$$

where TNR_j^{target} represent the target total net revenue (after tax) desired to be gleaned by the L_j agencies (opened at time 0) over time period T_j, in order to realize the goal that the cost basis of capital at time T_j is not less than $CA_j^{taget}(T_j)$.

Notice that time-period T_j has been defined as a kind of “value-based time limit (Chang and Chen, 2007),” as the service provider believes a certain degree of value-loss would be generated, if the time required to earn TNR_j^{target} for each country j is out of T_j. Instead, a certain degree of value-loss may be generated if the desire of the service provider—the cost basis of capital at time T_j reaches $CA_j^{taget}(T_j)$ for each country $j = 1,2,...,N$. Such a concept of value-loss may be predicated about the loss of market share or return of invested capital resulted from the entry of potential competitors or an increase in cost basis of capital of existing competitors. As a result finding a capital budgeting and commodity pricing policy so that a certain objective is optimized is one of most important decisions. Here we assume that the service firm offers multiple commodities but the production variable cost (per unit) relating to any type of commodity offered is the same as others, and thus the same pricing policy is made for all commodities offered.

Let f_j be the expected time required to earn TNR_j^{target} by those agencies planned to be opened in country j at time 0, then the time horizon BF_j which is defined as $T_j - f_j$ may be viewed as the buffer time to feed the variation of return of invested capital in country j. If there exists a capital budgeting policy so that all of target total net revenues (TNR_j^{target}) are expectedly earned within their associated value-based time limits (T_j), one may further find a solution to simultaneously maximize the buffer times in regards to each of target total net revenues. On the contrary, if there does not exist such a capital budgeting solution as described above, one may further find a solution to maximize this investment program' expected overall performance within those value-based time limits, with which a service provider is concerned. For simplicity, the former is called here the maximum buffer time (MBT) model and the latter the maximum achievement (MA) model. Consequently, it is assumed here that it is difficult to evaluate the variance of the return of invested capital, thus, a service provider may benefit by solving a MBT model. Restated, the purpose of MBT-modeling is to find a reliable capital budgeting and commodity pricing policy to ensure that each TNR_j^{target} is gleaned within its associated value-based time limit, T_j. It is not possible that one can find a solution of MBT model so that $BF_j \geq 0$ for all j if the expected time required to glean all of target total net revenues is more than the longest value-based time limit in regards to this N-country marketing-seeking program. In such a case a service provider may benefit from solving a MA model. Here we are only interested in the examination of a MBT model because the scope of a MA model is out of the purpose of this paper. Let p_j be the commodity pricing for country j and $Z_{jlt}(p_j, c_j)$ the return per unit time at time t to agency $l = 1,2.\cdots, L_j$ under given (p_j, c_j),we can see that

$$\sum_{l=1}^{L_j} \int_0^{f_j} (1-\alpha_j) Z_{jlt}(p_j, c_j) dt = TNR_j^{target},\ j = 1,2,\cdots, J \quad (8)$$

where c_j is the substitution of $c_j(0)$ and α_j denotes the tax rate in country j.

The MBT model is a type of maxmin (maximize minimum) format, i.e., to find an optimal (p_j, c_j) to maximize

$$BF^{\min} = \text{minimum } \{ BF_j = T_j - f_j, \quad j = 1,2,..., N \} \tag{9}$$

In order to further formulate the MBT model, one parameter, say c_j^*, is defined as below.

$$c_j^* = \min\left\{ \tilde{c}_j : \sum_l \int_0^{T_j} Z_{jlt}(p_j^*, \tilde{c}_j)dt \geq \underset{(p_j, c_j)}{\text{maximize}} \left\{ \sum_l \int_0^{T_j} Z_{jlt}(p_j, c_j)dt \right\} \right\} \tag{10}$$

where p_j^* denotes the optimal commodity pricing in country j.

Notice that with the use of c_j^*'s definition in (10), we may further define the value of $CIFP_j^*(0)$ in (2) as

$$CIFP_j^*(0) = L_j c_j^* \tag{11}$$

With the use of the results of (1)-(11), the MBT model may be initially formulated as follows:

$$\underset{(p_j, c_j)}{\text{maximize}} \ BF^{\min} = \text{minimum} \left\{ BF_j, \ j = 1,2,\cdots, N \right\} \tag{12a}$$

Subject to

$$BF_j \leq T_j - f_j, \ \forall j \tag{12b}$$

$$f_j \leq T_j, \forall j \tag{12c}$$

$$\sum_{l=1}^{L_j} \int_0^{f_j} (1 - \alpha_j) Z_{jlt}(p_j, c_j)dt \geq TNR_j^{target}, \ \forall j \tag{12d}$$

$$TNR_j^{target} = S_j(T_j) - L_j \xi_j c_j, \ \forall j \tag{12e}$$

$$\sum_{j=1}^{N} L_j \cdot c_j \le B_0 \tag{12f}$$

$$c_j^l \le c_j \le c_j^*, \ \forall j \tag{12g}$$

where c_j^l denotes the minimal amount of CIFP that the service provider hopes to expend in each agency located in country j, and notation $S_j(T_j)$ is the substitution of $L_j \xi_j c_j^*(T_j)$.

Specially, we must emphasis that in the case where $T_j = T_0$ for all j, the MBT model is equivalent to find an optimal (p_j, c_j) to minimize

$$f = \text{maximum}\left\{f_j,\ j = 1,2,\cdots,N\right\} \tag{13}$$

Such a model is referred to as a minimum makespan (MM) model in general.

Similarly, the MM model may be formulated as follows:

$$\underset{(p_j,c_j)\in\Omega}{\text{minimize}}\ f = \text{maximum}\left\{f_j,\ j = 1,2,\cdots,N\right\} \tag{14}$$

where Ω is the solution space in which each (p_j, c_j) solution satisfied (12b)-(12f).

3. Analytical Results

In order to obtain analytical results of the MBT model, it is necessary to make assumption about the form of $Z_{jlt}(p_j, c_j)$. Specially, we assume for each pair (p_j, c_j) that :

A1. $Z_{jlt}(p_j, c_j) = Z_j(p_j, c_j)$, if $t \le T_j$, which means the return per unit time is independent of time before time period T_j has elapsed as well as the profit structure to each agency is independent of what is an agency 's name.

A2. $Z_j(p_j, c_j) = [p_j - a_j(c_j)] \cdot K_j(c_j) d_j(p_j) - M_j(c_j) - O_j$ over $c_j \in [c_j^l, \infty)$

where

$K_j(c_j)$ = Number of potential customers who would purchase the commodities offered in each agency located in country j under investing amount of c_j in CIFP;

$d_j(p_j)$ = Demand rate (per unit time) for each customer while commodity pricing is p_j dollars;

$a_j(c_j)$ = Average variable cost per unit product under investing amount of c_j in CIFP;

$M_j(c_j)$ = Fixed cost (or ownership cost) per unit time for each agency in country j in regards to maintain the productivity and consistent quality under investing amount of c_j in CIFP;

O_j = Fixed cost (or ownership cost) per unit time for each agency in country j in regards to maintain the service quality under investing amount of $\overline{CIFD_j^*}(0)$ in CIFD.

A3. Demand rate $d_j(p_j)$ is the strictly decreasing linear function over the interval $[p_j^l, p_j^u]$, and it can be expressed as following form:

$$d_j(p_j) = \begin{cases} d_j^0, & p_j \le p_j^l, \\ d_j^0 - \delta_j(p_j - p_j^l), & p_j^l \le p_j \le p_j^u, \\ 0, & p_j \ge p_j^u, \end{cases}$$

where, $\delta_j = \dfrac{d_j^0}{p_j^u - p_j^l}$.

A4. Number of potential customers, $K_j(c_j)$, is the strictly increasing linear function over the interval $[c_j^l, \tilde{c}_j^u]$, and it can be expressed as following form:

$$K_j(c_j)=\begin{cases} K_j^l+\varphi_j\cdot(c_j-c_j^l), & \text{if } c_j^l\le c_j\le \tilde{c}_j^u \\ K_j^u, & \text{if } c_j\ge \tilde{c}_j^u \end{cases}$$

where c_j^l denotes the minimal amount of CIFP that the service provider hopes to expend in each agency, $\varphi_j=\dfrac{K_j^u-K_j^l}{\tilde{c}_j^u-c_j^l}$.

A5. Average variable cost per unit product, $a_j(c_j)$, is the strictly decreasing linear function over the interval $[c_j^l,\hat{c}_j^u]$, and it can be expressed as following form:

$$a_j(c_j)=\begin{cases} a_j^l-\pi_j\cdot(c_j-c_j^l), & \text{if } c_j^l\le c_j\le \hat{c}_j^u \\ a_j^u, & \text{if } c_j\ge \hat{c}_j^u \end{cases}$$

where, $\pi_j=\dfrac{a_j^l-a_j^u}{\hat{c}_j^u-c_j^l}$.

A6. Fixed cost per unit time, $M_j(c_j)$, is the strictly increasing exponential function over the interval $[c_j^l,\infty)$, and it can be expressed as following form:

$$M_j(c_j)=\theta_j[\exp(\beta_j c_j)-1],\ \ \theta_j,\beta_j>0$$

Notice that A 2 means that production policy for each agency is make-to-order and the lead time of satisfying a customer's demand is negligible under investing amount of $c_j, c_j\in[c_j^l,\infty)$, and thus the production capacity is sufficient for satisfying demand per unit time. In addition, it is also means that the demand rate of each customer depends on the price paid about attaining a product; however, the number of customer would depend on the commodity quality offered.

According to our assumptions, (12d) becomes

$$f_j = \frac{TNR_j^{target}}{L_j \cdot Z_j(p_j, c_j) \cdot (1-\alpha_j)}, j = 1,2,\cdots,N \tag{15}$$

Further, we assume that $T_j = T_0$ for all j. Base on above, it follows that the proposed MBT model will be equivalent to the following MM (minimum makespan) model:

$$\text{minimize } f = \max\left\{ f_j = \frac{TNR_j^{target}}{L_j \cdot Z_j(p_j, c_j) \cdot (1-\alpha_j)}, j = 1,2,\cdots,N \right\} \tag{16a}$$

Subject to

$$Z_j(p_j, c_j) = [p_j - a_j(c_j)] \cdot K_j(c_j) d_j(p_j) - M_j(c_j) - O_j \tag{16b}$$

$$f_j \le T_0, \forall j \tag{16c}$$

and (12e), (12f), (12g).

In order to solve this MM model, it has to find a legal policy (p_j, c_j) with the largest reward rate corresponding to investing in country j. That is, to solve the following problem:

$$\underset{p_j, c_j}{\text{maximize}}\, Z_j(p_j, c_j) = [p_j - a_j(c_j)] \cdot K_j(c_j) d_j(p_j) - M_j(c_j) - O_j \tag{17}$$

Lemma 1 Letting $p^*_{j|c_j}$ denote a pricing solution under given c_j corresponding to $\frac{\partial Z_j(p_j, c_j)}{\partial p_j} = 0$. Then $p^*_{j|c_j}$ has the following closed form:

$$p^*_{j|c_j} = \frac{d_j^0 + \delta_j \cdot a_j(c_j) + \delta_j \cdot P_j^l}{2\delta_j}.$$

Proof: Taking the first partial derivative of Z_j corresponding to p_j over the domain $[p_j^l, p_j^u]$, we have

$$\frac{\partial Z_j}{\partial p_j} = K_j(c_j)\cdot d_j^0 - 2\delta_j \cdot K_j(c_j)\cdot p_j + \delta_j \cdot p_j^l \cdot K_j(c_j) + \delta_j \cdot a_j(c_j)\cdot K_j(c_j)$$

Let $\frac{\partial Z_j}{\partial p_j} = 0$, the result of this lemma is obtained.

Theorem 1 For the demand rate function as (A3) corresponding to each buyer, the reward rate function $Z_j(p_j, c_j)$ is concave over $p_j \in [p_j^l, p_j^u]$ under given a certain value of c_j. Also, the optimal pricing solution whenever c_j is given is $\frac{d_j^0 + \delta_j \cdot a_j(c_j) + \delta_j \cdot P_j^l}{2\delta_j}$.

Proof: Taking the second partial derivative of Z_j corresponding to p_j over the domain $[p_j^l, p_j^u]$, we have $\frac{\partial^2 Z_j}{\partial p_j^2} = -2\delta_j \cdot K_j(c_j)$. Clearly, the results of this theorem are obtained from the result of Lemma 1 and the fact that $\left.\frac{\partial^2 Z_j}{\partial p_j^2}\right|_{p_j = p^*_{j|c_j}} < 0$.

In the practice, the parameters $\tilde{c}_j^u$ (in A4) and $\hat{c}_j^u$ (in A5) maybe are not equal; however, in this paper we are only interested in the case that $\tilde{c}_j^u = \hat{c}_j^u = c_j^u$.

According to above and the result of Theorem 1, we see that

$$Z_j(p^*_{j|c_j}, c_j) = \begin{cases} [\frac{d_j^0 + \delta_j[p_j^l - a_j^l + \pi_j \cdot (c_j - c_j^l)] + \pi_j \delta_j \cdot (c_j - c_j^l)}{2\delta_j}] \cdot [K_j^l + \varphi_j \cdot (c_j - c_j^l)] \\ \cdot [\frac{d_j^0 + \delta_j[3p_j^l + a_j^l] - \pi_j \delta_j (c_j - c_j^l)}{2}] - \theta_j\{\exp(\beta_j c_j) - 1\} - O_j, \qquad c_j \in [c_j^l, c_j^u] \\ [\frac{d_j^0 + \delta_j \cdot (p_j^l - a_j^u)}{2\delta_j}] \cdot [K_j^u] \cdot [\frac{d_j^0 + \delta_j[3p_j^l + a_j^u]}{2}] - \theta_j\{\exp(\beta_j c_j) - 1\} - O_j, \quad c_j \in [c_j^u, \infty) \end{cases} \tag{18}$$

Define

$$\tilde{Z}_j(c_j) = \begin{cases} [\frac{d_j^0 + \delta_j[p_j^l - a_j^l + \pi_j \cdot (c_j - c_j^l)] + \pi_j\delta_j \cdot (c_j - c_j^l)}{2\delta_j}] \cdot [K_j^l + \varphi_j \cdot (c_j - c_j^l)] \\ \cdot [\frac{d_j^0 + \delta_j[3p_j^l + a_j^l] - \pi_j\delta_j(c_j - c_j^l)}{2}], \end{cases} \tag{19a}$$

$$\hat{Z}_j(c_j) = \theta_j\{\exp(\beta_j c_j) - 1\} - O_j \tag{19b}$$

Then (18) becomes

$$Z_j(p^*_{j|c_j}, c_j) = \begin{cases} \tilde{Z}_j(c_j) - \hat{Z}_j(c_j), & c_j \in [c_j^l, c_j^u] \\ [\frac{d_j^0 K_j^u + \delta_j K_j^u \cdot (p_j^l - a_j^u)}{2\delta_j}] \cdot \cdot [\frac{d_j^0 + \delta_j[3p_j^l + a_j^u]}{2}] \\ -\theta_j\{\exp(\beta_j c_j) - 1\} - O_j, & c_j \in [c_j^u, \infty) \end{cases} \tag{20}$$

This yields

$$\begin{aligned} \tilde{Z}_j' = {} & [d_j^0 - \delta_j a_j^l + \delta_j \pi_j (c_j - c_j^l)] \cdot [\frac{\pi_j K_j^l + \pi_j \varphi_j \cdot (c_j - c_j^l)}{2} + \frac{\varphi_j p_j^l}{2}] \\ & + \frac{\varphi_j}{4\delta_j} \cdot [d_j^0 - \delta_j a_j^l + \delta_j \pi_j (c_j - c_j^l)]^2 + \frac{\pi_j \delta_j p_j^l}{2} \cdot [K_j^l + \varphi_j \cdot (c_j - c_j^l)] \\ & + \frac{(p_j^l)^2 \cdot \delta_j \cdot \varphi_j}{4} \end{aligned} \tag{21a}$$

$$\hat{Z}_j' = \alpha_j \beta_j \cdot \exp(\beta_j c_j) \tag{21b}$$

By the same token, we have

$$\tilde{Z}_j'' = \{\frac{\pi_j^2 \delta_j}{2} \cdot [K_j^l + \varphi_j \cdot (c_j - c_j^l)\} + \pi_j \varphi_j \cdot [d_j^0 - \delta_j a_j^l + \delta_j \pi_j \cdot (c_j - c_j^l) + \delta_j p_j^l] \tag{22a}$$

$$\hat{Z}_j'' = \alpha_j \beta_j^2 \cdot \exp(\beta_j c_j) \tag{22b}$$

Lemma 2: If $\tilde{Z}_j''(c_j^{l+}) < \hat{Z}_j''(c_j^{l+})$, then $Z_j(c_j)$ is concave over $c_j \in [c_j^l, c_j^u]$, where c_j^{l+} is a neighborhood of c_j^l and $c_j^{l+} > c_j^l$.

Proof: Clearly, we see that $\tilde{Z}_j''(c_j)$ is strictly increasing linear function and $\hat{Z}_j''(c_j)$ is the strictly increasing convex function over $c_j \in [c_j^l, c_j^u]$ by (22a) and

(22b). Thus, by the fact that $\tilde{Z}_j''(c_j^{l+}) < \hat{Z}_j''(c_j^{l+})$, we have that $\tilde{Z}_j''(c_j) < \hat{Z}_j''(c_j)$ over $c_j \in [c_j^l, c_j^u]$. That is, $Z_j(p^*_{j|c_j}, c_j)$ is concave over $c_j \in [c_j^l, c_j^u]$.

Lemma 2:

(1) If $\tilde{Z}_j'(c_j^{l+}) > \hat{Z}_j'(c_j^{l+})$, $\tilde{Z}_j'(c_j^{u-}) < \hat{Z}_j'(c_j^{u-})$, and $\tilde{Z}_j''(c_j^{l+}) < \hat{Z}_j''(c_j^{l+})$; then $Z_j^{'}(p^*_{j|c_j}, c_j) = 0$ (i.e., (18a)-(18b)=0) has the solution over $c_j \in (c_j^l, c_j^u)$, this solution is the global maximum, where c_j^{u-} is a neighborhood of c_j^u and $c_j^{u-} < c_j^u$.

(2) If $\tilde{Z}_j'(c_j^{l+}) > \hat{Z}_j'(c_j^{l+})$, $\tilde{Z}_j'(c_j^{u-}) > \hat{Z}_j'(c_j^{u-})$, and $\tilde{Z}_j''(c_j^{l+}) < \hat{Z}_j''(c_j^{l+})$; then c_j^u is the global maximum.

Proof: (1) By the results of $\tilde{Z}_j'(c_j^{l+}) > \hat{Z}_j'(c_j^{l+})$, $\tilde{Z}_j'(c_j^{u-}) < \hat{Z}_j'(c_j^{u-})$, we have that Z_j is increasing at $c_j = c_j^{l+}$ and decreasing at $c_j = c_j^{u-}$. Also, by the results of $\tilde{Z}_j''(c_j^{l+}) < \hat{Z}_j''(c_j^{l+})$ and Lemma 2, we get $Z_j^{'}(p^*_{j|c_j}, c_j) = 0$ has a solution and this solution is local maximum over $c_j \in [c_j^l, c_j^u]$. Further, by our assumptions and (18), we get $Z_j(p^*_{j|c_j}, c_j) \le Z_j(p^*_{j|c_j}, c_j^u)$ over $c_j \in [c_j^u, \infty)$, thus this solution is also the global maximum over $c_j \in [c_j^l, \infty)$.

(2) The proof is analogous to the proof of (1) of this lemma, thus omitted.

Theorem 2: If $\tilde{Z}_j'(c_j^{l+}) > \hat{Z}_j'(c_j^{l+})$ and $\tilde{Z}_j''(c_j^{l+}) < \hat{Z}_j''(c_j^{l+})$, then $Z_j(p^*_{j|c_j}, c_j)$ is a strictly increasing concave function either over $c_j \in [c_j^l, \tilde{c}_j^*]$ or over $c_j \in [c_j^l, c_j^u]$, where $\tilde{c}_j^*$ is the solution of $Z_j^{'}(p^*_{j|c_j}, c_j) = 0$.

Proof: The results of this theorem are straightforwardly obtained by the Lemma 2.

4. Solution Method

4.1 Piecewise-linear Approximation

According to the result of Theorem 1, (16a) may be rewritten as

$$\text{minimize } f = \max\left\{ f_j = \frac{TNR_j^{target}}{Z_j(p^*_{j|c_j}, c_j)\cdot(1-\alpha_j)},\ j = 1,2,\cdots,J \right\} \tag{23}$$

where

$$p^*_{j|c_j} = \frac{d_j^0 + \delta_j P_j^l + \delta_j \cdot a_j(c_j)}{2\delta_j}$$

Now we consider the case that $\tilde{Z}_j'(c_j^{l+}) > \hat{Z}_j'(c_j^{l+})$ and $\tilde{Z}_j''(c_j^{l+}) < \hat{Z}_j''(c_j^{l+})$, then according to Theorem 2, we see that $\tilde{c}_j^*$ is the solution of (24) by (22a) and (22b).

$$\begin{cases} [\dfrac{d_j^0 + \delta_j[p_j^l - a_j^l + \pi_j\cdot(c_j - c_j^l)] + \pi_j\delta_j\cdot(c_j - c_j^l)}{2\delta_j}]\cdot[K_j^l + \varphi_j\cdot(c_j - c_j^l)] \\ \cdot[\dfrac{d_j^0 + \delta_j[3p_j^l + a_j^l] - \pi_j\delta_j(c_j - c_j^l)}{2}] - \theta_j\beta_j\exp(\beta_j c_j) = 0 \end{cases} \tag{24}$$

Equation (24) can be solved by using the well-known Newton-Raphson method. Also, we have that $c_j^* = \tilde{c}_j^*$ or c_j^u, where c_j^* is defined by (10). After obtaining the values of c_j^* for all j, we see that $Z_j(p^*_{j|c_j}, c_j)$ is a strictly increasing concave function over $c_j \in [c_j^l, c_j^*]$ (by Theorem 2). Thus, the technique of piece-wise linear approximation may be employed to transform the nonlinear fashion of $Z_j(p^*_{j|c_j}, c_j)$ into an approximately linear form(as depicted in Figure 1). Indeed, if we take K_j breaking points from interval $[c_j^l,\ c_j^*]$, noted by $r_{j(k)}, k = 0,1,\cdots,K_j$, then there exist some $c_{j(k)}$, $0 \le c_{j(k)} \le r_{j(k)} - r_{j(k-1)}$, $r_{j(0)} = c_j^l$, $r_{j(K_j)} = c_j^*$, so that $Z_j(p^*_{j|c_j}, c_j)$ may be rewritten as

$$Z_j(p^*_{j|c_j}, c_j) \approx Z_j(p^*_{j|r_{j(0)}}, r_{j(0)}) + \sum_{k=1}^{K_j} \rho_{j(k)}\cdot c_{j(k)},\ \forall j \tag{25}$$

where

$$\rho_{j(k)} = \frac{Z_j(p^*_{j|r_{j(k)}}, r_{j(k)}) - Z_j(p^*_{j|r_{j(k-1)}}, r_{j(k-1)})}{r_{j(k)} - r_{j(k-1)}},.$$

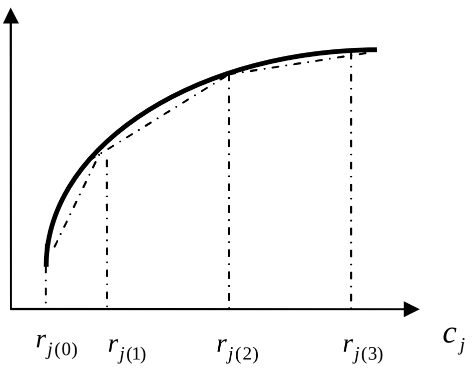

Figure 1 Piecewise linear approximation

Substituting (25) for $Z_j(p^*_{j|c_j}, c_j)$ in (24)，the proposed MM model may be rewritten as (26a)-(26e).

$$\text{minimize } f = \max\left\{ f_j = \frac{TNR_j^{\text{target}}}{L_j \cdot \left[Z_j(p^*_{j|r_{j(0)}}, r_{j(0)}) + \sum_{k=1}^{K_j} \rho_{j(k)} \cdot c_{j(k)} \right] \cdot (1-\alpha_j)}, j = 1,2,\cdots,N \right\} \tag{26a}$$

Subject to

$$TNR_j^{\text{target}} = S_j(T_j) - L_j \xi_j (r_{j(0)} + \sum_k c_{j(k)}),\ \forall j \tag{26b}$$

$$\sum_{j=1}^{J} L_j \cdot \left[r_{j(0)} + \sum_{k=1}^{K_j - 1} c_{j(k)} \right] \le B_0 \tag{26c}$$

$$t_j \le T_0 \tag{26d}$$

$$0 \le c_{j(k)} \le r_{j(k)} - r_{j(k-1)},\ \forall j,\ k \tag{26e}$$

4.2 Fractional Programming Method

Because (26a) has the fractional characteristic, we can further use the Fractional

Programming method to transform (26a) into a linear form. Indeed, (26a) may be rewritten as

$$\text{maximize}\frac{1}{f}=\min\left\{\frac{1}{f_j}=\frac{L_j\cdot\left[Z_j(p^*_{j|r_{j(0)}},r_{j(0)})+\sum_{k=1}^{K_j}\rho_{j(k)}\cdot c_{j(k)}\right]\cdot(1-\alpha_j)}{TNR_j^{\text{target}}},\forall j\right\} \quad (27)$$

Moreover, if we let

$$x_{j(k)}=\frac{c_{j(k)}}{S_j(T_j)-L_j\xi_j\left\{r_{j(0)}+\sum_k c_{j(k)}\right\}},$$

$$\lambda_j=\frac{1}{S_j(T_j)-L_j\xi_j\left\{r_{j(0)}+\sum_k c_{j(k)}\right\}},$$

The proposed MM model may be rewritten as

Objective: maximize y (28a)

Subject to

$$y\le y_j,\forall j \quad (28b)$$

$$y_j=L_j\left\{[Z_j(p^*_{j|r_{j(0)}},r_{j(0)})\lambda_j+\sum_{k=1}^{K_j}\rho_{j(k)}x_{j(k)}]\cdot(1-\alpha_j)\right\},\forall j \quad (28c)$$

$$\left(S_j(T_j)-L_j\xi_j r_{j(0)}\right)\lambda_j-\sum_{k=1}^{K_j}L_j\xi_j x_{j(k)}=1,\forall j \quad (28d)$$

$$\sum_{j=1}^{J}L_j\cdot\left[r_{j(0)}+\sum_{k=1}^{K_j}\frac{x_{j(k)}}{\lambda_j}\right]\le B_0 \quad (29e)$$

$$T_0\cdot y_j\ge 1,\forall j \quad (28f)$$

$$x_{j(k)}-(r_{j(k)}-r_{j(k-1)})\cdot\lambda_j\le 0,\quad \forall j,k \quad (28g)$$

$$x_{j(k)}\ge 0,\quad \forall j,\ k \quad (28h)$$

$$\lambda_j\ge 0,\quad \forall j \quad (28i)$$

where $y=1/f$; $y_j=1/f_j$, $\forall j$

4.3 Weighting Method

Because (28e) still possesses the fractional term, $\frac{x_{j(k)}}{\lambda_j}$, to solve above model is still difficult.

Rather, owing to the fact that

$$\sum_{j=1}^{J}\sum_{k=1}^{K_j}\frac{L_j x_{j(k)}}{\lambda_j} \le B_0 - \sum_{j=1}^{J} L_j r_{j(0)} \ , \qquad (29)$$

we see that there exist some weights $w_j \in (0,1)$ so that

$$\sum_j w_j = 1 \qquad (30)$$

$$\sum_{k=1}^{K_j}\frac{L_j x_{j(k)}}{\lambda_j} = w_j (B_0 - \sum_{j=1}^{J} L_j r_{j(0)}), \ \forall j \qquad (31)$$

Collecting the terms in (29), we see that

$$\sum_{k=1}^{K_j} L_j x_{j(k)} = w_j (B_0 - \sum_{j=1}^{J} L_j r_{j(0)})\lambda_j , \ \forall j \qquad (32)$$

.

Based on above, a simple weighting method is given as follows:

Step 1: Add the variables, $w_j, \forall j$ so that $\sum w_j = 1$;

Step 2: Substitute (31) for (28e).

Step 3: Use OR software (e.g., LINGO 8.0) which is capable of solving the optimization models which include the simple quadratic constraints like as (32) to solve the proposed MM model.

After obtaining the values of $x_{j(k)}$ and λ_j, it is easy to compute c_j by the following formula.

$$c_j = r_{j(0)} + \sum_{k=1}^{K_j - 1} \frac{x_{j(k)}}{\lambda_j}, \forall j \tag{33}$$

In short, the proposed solution method to find an optimal commodity pricing and capital investment policy for the MM model can be summarized as follows:

(i) Use (24) to find the optimal budgeting policy $(c_1^*, \ldots, c_j^*, \ldots, c_N^*)$, which maximize the reward rate function $Z_j(p^*_{j|c_j}, c_j)$ for all countries j;

(ii) Use piecewise-linear approximation technique to transform the concave curve of $Z_j(p^*_{j|c_j}, c_j)$ into a linear form over the domain $[c_j^l, c_j^*], \forall j$;

(iii) Use fractional programming technique to transform the fractional type of objective function (as (26a)) into the linear form (as (28a)-(28c)),

(iv) Use the simple weighting method described above to transform the fractional type of constraints (as (28e)) into the a simple quadratic form (as (30) and (32));

(v) Use (33) to obtain optimal capital investment policy $(c_1^{**}, \ldots, c_j^{**}, \ldots, c_N^{**})$, which minimize the time required to earn the target total net revenues for all countries j;

(vi) Compute $(p^*_{1|c_1^{**}}, \ldots, p^*_{j|c_j^{**}}, \ldots, p^*_{N|c_N^{**}})$ to obtain the optimal pricing policy by the following formula: $p^*_{j|c_j^{**}} = \dfrac{d_j^0 + \delta_j P_j^l + \delta_j \cdot a_j(c_j^{**})}{2\delta_j}$.

5. Illustrative Example

Consider the well-known Chinese food X has a parent company which exists in Taiwan. However, for the market-seeking purpose, the firm intends to expand its business to Asia market with wholly owned based FDI. Assume six countries are chosen to invest a certain amount in capital at the initial investment phase and they are coded by number 1 to 6. Further, assume that each country would have a large increase in the demand rate when 25 unit times have elapsed, i.e., $T_0 = 25$. Also, each country will only open a store, i.e., $L_j = 1, \forall j$. Other parameters are stated as Table 1. According to Table 1 and (22), the values of c_j^* are depicted as Table 2.

Moreover, five breaking points are given in Table 3 and the segment slopes for all piecewise-linear approximation are depicted as Table 4. Finally, by using Lingo 8.0, we find the optimal capital investment policy $(c_1^{**}, \ldots, c_j^{**}, \ldots, c_N^{**})$ and optimal pricing policy $(p^*_{1|c_1^{**}}, \ldots, p^*_{j|c_j^{**}}, \ldots, p^*_{N|c_N^{**}})$ stated as Table 5. Based on Table 5, the expected time required to earn the total net revenues for all countries is 22.9865 unit times, which approaches 23 unit times.

6. Concluding Remarks

A multi-site locations expansion model has been proposed to find an optimal commodity pricing and capital distribution scheme for services internationalization. Having found some properties of the model, we proposed a solution method consisting of the techniques of piece-wise linear approximation, linear fractional programming and weighting method. The results of this paper are quite useful for service firms. Specially, the international markets planning expansion on such business options as fast food, steak restaurant, and café shops, and so on. In this paper we only examined the case that there exists a capital budgeting solution so that all of target total net revenues are expectedly earned within their associated value-based time limits (i.e. MBT model), thus further effort may focus on developing a maximum achievement model which aims to find a solution to maximize this investment program' expected overall performance within those value-based time limits, with which a service provider is concerned.

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Appendix:

Table1: Related parameters of DSAS-MM model

country j	1	2	3	4	5	6
c_j^l	800	700	700	900	800	900
c_j^u	3000	2500	2800	3500	2900	2800
p_j^l	0.01	0.01	0.02	0.01	0.01	0.01

p_j^u	0.30	0.35	0.33	0.31	0.32	0.29
K_j^l	500	500	500	500	700	480
K_j^u	1500	1400	1900	1800	1700	1400
a_j^l	0.090	0.08	0.070	0.090	0.075	0.060
a_j^u	0.06	0.04	0.04	0.04	0.04	0.03
d_j^0	7	4	5	6	5	6
d_j^u	0	0	0	0	0	0
α_j	30	30	35	20	20	30
β_j	0.0010	0.0010	0.0010	0.0011	0.0011	0.0010
ξ_j	1	1	1	1	1	1
$S_j(T_j)$	4550	2900	2100	5600	3800	3400

Table 2: values of c_j^l and c_j^*

country j	1	2	3	4	5	6
c_j^l	800	700	700	900	800	900
c_j^*	2180	2000	2200	2200	2150	2210

Table 3: Invested Capital and reward rate: $r_{j(k)}$ and $Z_j(p^*_{j|r_{j(k)}}, r_{j(k)})$

country j	$k=0$		$k=1$		$k=2$		$k=3$		$k=4$	
1	800	154	1000	179	1400	222	1950	267	2180	274
2	700	146	1150	188	1570	216	1700	226	2000	233
3	700	124	1050	175	1500	227	1760	251	2200	258
4	900	160	1100	188	1590	234	1950	259	2200	267
5	800	179	1050	208	1590	252	1800	263	2250	266
6	900	140	1150	178	1500	209	1800	230	2210	232

Table 4: Segment slopes for all piecewise-linear approximation

slope / country j	$\rho_{j(1)}$	$\rho_{j(2)}$	$\rho_{j(3)}$	$\rho_{j(4)}$
1	0.0946	0.0659	0.0502	0.0313
2	0.0750	0.0522	0.0395	0.0527
3	0.0706	0.0574	0.0417	0.0286
4	0.0896	0.0675	0.0445	0.0346
5	0.0942	0.0775	0.0643	0.0300
6	0.0786	0.0552	0.0375	0.0264

Table 5 Results of this illustrative example

Objective Value f ：22.9865

country j	$c^*_{j(1)}$	$c^*_{j(2)}$	$c^*_{j(3)}$	$c^*_{j(4)}$	c^l_j	c^{**}_j	$p^*_{j\|c^{**}_j}$	f_j
1	200	400	494	0	800	1894	0.137	22.9865
2	450	420	130	47	700	1746	0.198	22.9865
3	350	282	0	0	700	1332	0.185	22.9865
4	200	439	0	0	900	1539	0.188	22.9865
5	250	505	0	0	800	1555	0.186	22.9865
6	250	350	300	131	900	1931	0.161	22.9865