MARGINAL CONDITIONAL STOCHASTIC DOMINANCE BETWEEN VALUE AND GROWTH

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ABSTRACT

Marginal Conditional Stochastic Dominance (MCSD) is an extension of the second order stochastic dominance that considers the joint nature of return distributions. It is a useful tool for examining marginal dominance of one asset to another conditionally to a given market return distribution for all risk-averse investors. MCSD is superior to conventional market models in that it requires no modeling specification and is distributional free. Although the size and value effect of equity portfolio performance has been well documented, most of analysis relies on statistical regression description and/or linear factor models. This manuscript applies MCSD to re-exanimate the size/value effects for international equity markets. The empirical MCSD test reveals that U.S. value stocks outperformed the market and dominated growth stocks for the post 1975 period. However, the phenomenon of *value* over *growth* is generally insignificant in markets around the world, and it varies with different valuation criteria.

Keywords: Value, Size, Stochastic Dominance, and Portfolio Selection.

JEL Classification: G11, G14

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I. INTRODUCTION

Buying stocks with high book to market (B/M) ratios, so-called value stocks, will produce returns that outperform the market. This value strategy has been documented by Chan, Hamao, and Lakonishok (1991), DeBondt and Thaler (1985, 1987), Fama and French (1992, 1996, 1998), Lakonishok, Shleifer and Vishny (1994), and Haugen and Baker (1996). However, despite pervasive evidence of value effects, there remains debate on this issue. Some argue that empirical evidence supporting the value effect may be simply a data snooping bias in that the anomalies are sample specific results that are unlikely to be observed out of sample.[1] Other researchers argue that the higher average returns on small firm value stocks are compensation for risk.[2] That is, the tendency of value stocks to outperform growth stocks is not an anomaly. It can be viewed as a risk factor, in equilibrium, priced in addition to the traditional CAPM type systematic risk.[3]

Recently, debate has also centered on the source of the value-growth effect. One explanation is that investors overreact to performance and assign irrationally low values to distress stocks and irrationally high values to growth stocks. When the overreaction is corrected, distressed firms experience high stock returns and growth firms experience low stock returns.[4] In contrast, Fama and French (1993, 1995, 1996, 1998) argue that the value premium is compensation for systematic risk. There is no evidence that average returns vary with firm size and B/M in a way that cannot be explained by risk loading, and there is no evidence that

[1] See MacKinlay (1995), Knez and Ready (1997) and Loughran (1997).
[2] See Brennan, Chordia, and Subrahmanyam (1998), Chan, Chen and Hsieh (1985), Chan, Karceski and Lakonishok (1998), Chen and Zhang (1998), and Dichev (1998).
[3] For example, Fama and French (1993, 1995, 1996, 1997, 1998) have proposed a three-factor model that is able to describe stock returns.
[4] Proponents of this view include De Bondt and Thaler (1987), Lakonishok, Shleifer and Vishny (1994), and Daniel and Titman (1997, 1998).

variation in risk loadings is uncompensated when it is unrelated to size and B/M.[5] Therefore, the risk model is perhaps the most appropriate approach to explain and/or analyze the value-growth effect.

Since a full description of expected returns with associated risk factors for stock portfolios must obviously be model specific, the potential problems of model misspecification or invalid modeling assumptions may provide unreliable results. Specifically, the linearity and symmetry of return distributions to permit portfolio separation is necessary for the validity of capital asset pricing models.[6] Modeling results could thus be misleading, if the return generating process of assets is non-linear and/or asymmetrically distributed. For ranking investment alternatives and/or portfolio performance results, Stochastic Dominance (SD) is superior to conventional capital asset models such as CAPM and APT in that it derives weak conditions for separation based on general probability distributions, is consistent with expected utility maximization, and places no restrictions on the class of investor utility functions.[7] Importantly, it requires no specification about the linearity of the return generating process. However, although SD is a general and powerful tool for rank ordering portfolios based on their risk and return trade-off without any asset pricing modeling specification, it involves serious pitfalls in portfolio analysis. Levy (1992) notes that SD performs well in applied economics and finance when the decision problem is preference for a single asset or policy. But in optimal portfolio selection, SD performs poorly in that one has to search through all possible combinations of

[5] To distinguish the risk model from the overreaction model, one must be able to find variation in size and B/M characteristics unrelated to risk loading. See Davis, Fama and French (1998).

[6] Portfolio separation is important and necessary for providing the equilibrium results of capital asset models. For example, the two-fund separation is necessary for CAPM [Sharpe (1964)], and the *N*-fund separation is critical for the APT model [Ross (1976)] . To obtain portfolio separation, either the utility function needs to be restricted [Cass and Stiglitz (1970)] or the return distribution is under certain restrictions. Ross (1978) explicitly demonstrates that for all risk averse utility functions, to permit portfolio separation, the return generating process must follow a linear structure. Ross' linear distribution separation is quite general in that it requires no specification about the form of utility function and that of the return distribution such as normality and/or elliptical distributions.

portfolios to find an efficient one. In addition, the SD is originally calculated by independently comparing the cumulative return distributions of assets without considering the joint nature of the assets' return distributions.[8] For portfolio performance analysis, however, it is important that a measure of performance be insensitive to the relative risk of each portfolio and the strength of the market (core portfolio) condition. Such a measure needs to adjust the portfolio's return by the amount of return that is attributable to the relative risk of the local portfolio, given the strength of the market (core) portfolio in the period that performance is evaluated. Conventional stochastic dominance rules unfortunately fail to adjust the ordering of assets' return distributions by changes in market conditions, and, consequently, the SD ordering is sensitive to market strength.

Marginal Conditional Stochastic Dominance (MCSD), developed originally by Shalit and Yitzhaki (1994), orders assets marginally and conditionally from a given portfolio.[9] MCSD theory is derived from the concept that, in the asset selection process, all risk-averse investors will prefer a particular option to another, given that they hold the rest of the portfolio. That is, investors can improve expected utility by marginally increasing the dominating portfolios at the expenses of the dominated ones. Shalit and Yitzhaki (2003) argue that not only the traditional Stochastic Dominance approach does not provide practical results as it involves an infinite number of pair-wise comparison of portfolios, but constructing dominating portfolios according to SD is bound to fail because one can always find a combination yielding higher expected returns. Therefore, rather than build an optimal portfolio, one could employ MCSD to determine whether a given portfolio belongs to the SD efficient set so that it is impossible to find an

[7] See Hadar and Russell (1969), Hanoch and Levy (1969) and Rothschild and Stiglitz (1970).
[8] For instance, asset *A* second-degree stochastically dominates asset *B* if and only if the twice-cumulated density function *(c.d.f.)* of *A* is not greater than that of *B* for all levels of returns. The comparison is stand-along and ignores the joint nature between individual assets and the overall market conditions.

alternative portfolio that is pair-wise preferred by all risk-averse investors. That is, instead of finding the entire SD efficient set, attention centers on whether a given portfolio belongs to the efficient set. Specifically, if the original portfolio is inefficient and/or not optimal, then one is able can find an alternative portfolio by marginally changing the allocation so that the new portfolio is superior in the eyes of every risk-averse investor.

This paper applies the MCSD technique to examine the anomaly of value/size effect. In financial theory, if the market is efficient, then the market portfolio should be in the efficient set and thus no alternative portfolio should dominate the market portfolio by reallocating its assets.[10] Thus, using MCSD to examine the existence of value/size anomaly is quite intuitive. Let the market (core) portfolio be decomposed by a set of mutually exclusive different B/M ratio and size sub-portfolios similar to those in Fama and French (1992, 1996, and 1998). [11] If there is at least one marginal dominance condition among the local portfolios, e.g. the local portfolio of low B/M (value) dominates that of the high B/M (growth), then, according to the theory of MCSD, the market portfolio is inefficient. From the viewpoint of all risk-averse investors, a superior portfolio can be formed by taking long positions in the marginally dominating shares of value stocks at the cost of shorting the marginally dominated shares of growth stocks. In brief, the test of anomaly focuses on the efficiency of the market portfolio by evaluating the marginal contribution of a local portfolio to the core portfolio. In addition, since MCSD ranks portfolios by comparing the conditional return distributions of portfolios with respect to the market return

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[9] Shalit and Yitzhaki (2003) have applied the MCSD to an asset allocation puzzle.

[10] An efficient set of portfolios is that inside the set, no portfolio is dominated by any other portfolio. If the return distribution of assets can be characterized by the first two moments of the distribution, efficient portfolios of risky and/or risk-free assets are located on the Capital Market Line (CML) in the mean-variance framework. The market portfolio is also located on the CML. In equilibrium, the market portfolio is optimal, and other combinations of assets should dominate the market portfolio.

[11] All stocks are separated into two size groups, small or big (S or B), based on the median size for all stocks concerned. Further, stocks are broken into three book to market equity (B/M) groups based on the break points for the bottom 30% (L), middle 40% (M) and top 30% (H) of the ranked values of B/M for the stocks in question.

distribution, unlike traditional SD, MCSD ordering results are insensitive to changes in market conditions. The MCSD approach is superior to conventional performance measurements, such as Sharpe (1966, 1994), Treynor (1966), and Jensen (1968), in that MCSD considers the entire joint distribution of assets and the market, not just summary statistics such as the mean, variance, and beta coefficients. Importantly, MCSD, unlike the traditional market models, does not rely on a linear return generating process and makes no assumption about the form of the underlying probability distribution. Chow (2001) developed a simple statistical test for MCSD and showed that it has the power to detect dominance for samples with more than 300 observations, and is robust under both homoskedasticity and heteroskedasticity.

The paper is organized as follows. Section II reviews the MCSD ranking rule and its statistical inference procedures. Using data provided by Kenneth French, section III provides an empirical analysis of MCSD. The results show that value portfolios do outperform growth portfolios and the market portfolio in U.S. markets. However, using international data, the dominance of value stocks over growth stocks does not appear to hold worldwide. Interestingly, international stock markets appear to have different value-growth effects using different valuation criteria including book to market (B/M), earnings to price (E/P), cash earnings to price (C/P), and dividend yield to price (D/P). Finally, section IV provides brief concluding remarks.

II. Marginal Conditional Stochastic Dominance Test

Suppose investors hold a diversified core-portfolio, and the core-portfolio can be decomposed into a set of *n* mutually exclusive sub-portfolios. For example, let the value weighted portfolio of all stocks be the core-portfolio, and it can be decomposed into a set of sub-portfolios according to different *value* and *size* criteria as suggested by Fama and French (1992).

The return of the core-portfolio can be written as $r_m = \sum_{p=1}^{n} w_p r_p$, where r_p is the return of the *p*-th sub-portfolio, and $\sum_{i=1}^{n} w_i = 1$. Assume that investors are maximizing their expected utility of returns. If the existing asset allocation of the core-portfolio is not optimal, investors will be able to improve their expected utility by increasing the holding of one sub-portfolio ***p*** by decreasing their position of another sub-portfolio ***q***. For instance, if anomalies such as the value/size effects do exist, a portfolio reallocation process implemented by active investment strategies, i.e., buying value stocks at the cost of selling growth stocks, will increase investor utility. That is, investors increase w_p and decrease w_q keeping the sum constant, so that

(1) $$dw_p + dw_q = 0$$

Shalit and Yitzhaki (1994) demonstrate that

Definition 1. *For all risk-averse and expected utility maximizing investors, given the existing market portfolio, the following condition ensures that investors prefer to increase holdings of portfolio k and decrease holdings of portfolio j:*

(2) $$\frac{d}{dw_p} E(u(W)) = E[u'(W)(r_p - r_q)] \geq 0,$$ [12] *where* $$W = 1 + \sum_{p=1}^{n} w_p r_p \ .$$

Shalit and Yitzhaki (1994) formulate the necessary and sufficient conditions, called MCSD, to ensure the inequality (2) in terms of concentration curves (ACCs), which are defined as the cumulative expected returns on a sub-portfolio conditional on the return on the core-portfolio.

[12] This is the standard Arrow (1970, p. 101) condition, but for random wealth.

Since the concept of ACC is less familiar to financial researchers, Chow (2001) re-formulates the MCSD conditions in a relatively simple framework as follows:

Theorem 1. Marginal Conditional Stochastic Dominance (MCSD). *For all risk-averse investors (u"<0), portfolio p marginally and conditionally dominates portfolio q such that* $E[u'(W)(r_p - r_q)] \geq 0$ *if and only if*

(3.1) $$\int_{-\infty}^{\tau_\rho^m}\int_{-\infty}^{\infty} r_p\, f(r_p, r_m)dr_m dr_p \geq \int_{-\infty}^{\tau_\rho^m}\int_{-\infty}^{\infty} r_q\, f(r_q, r_m)dr_m dr_q$$, or

(3.2) $$E\left(r_p - r_q \middle| r_m \leq \tau_\rho^m\right) \geq 0$$, or

(3.3) $$E\left((r_p - r_q)\, I(\tau_p^m)\right) \geq 0$$,

for all ρ, *where* $0 \leq \rho \leq 1$, *E is the expectation operator, and* $\tau_\rho^m = F_m^{-1}(\rho)$.[13] $F_m = \rho$ *is the cumulative density function of* r_m.[14] $I(\tau_p^m) = 1$, *if* $r_m \leq \tau_p^m$, *and* $I(\tau_p^m) = 0$, *otherwise.*

It is important to note that the inequality (3) is consistent with expected utility maximization without prior knowledge about individual utility functions and the underlying form of the return generating process of assets. Therefore, the MCSD rule is separated from an individual investor's utility and is also distribution-free. In addition, when $\rho = 1$, the inequality (3) is equivalent to the difference of mean returns between sub-portfolios *p* and *q*, respectively.[15] Shalit and Yitzhaki (1994) further showed that if a MCSD exists between two sub-portfolios, then the following inequality of Gini-risk adjusted means must hold:

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[13] Since $\int_{-\infty}^{\tau_p^m}\int_{-\infty}^{\infty} r_k\, f(r_k, r_m)dr_m dr_k = pE\left(r_k \middle| r_m \leq \tau_p^m\right)$, the inequality (4) also holds.

[14] Shalit and Yitzhaki (1994) apply the concept of Absolute Concentration Curve (ACC) often used in income inequality study to prove the necessary and sufficient conditions of (2). Chow (2001) explicitly shows that the expression of ACC inequality is equivalent to the conditional mean inequality.

[15] We assume that the distribution is continuous and monotony increase such that $\tau_\rho^m = \infty$ for $\rho = 1$.

Lemma 1. *<u>Necessary Condition of MCSD</u>. Let μ_p and μ_q be the mean returns of sub-portfolios p and q respectively. The conditions of positive (Gini risk adjusted) premium,*

(4.1) $$\mu_p - \mu_q \geq 0, \text{ and}$$

(4.2) $$\mu_p - \beta_p\Gamma_p \geq \mu_q - \beta_q\Gamma_q,$$

are necessary but insufficient to have a positive change in expected utility as referred by the inequality (2), *where β_p and β_q conventional beta coefficients of r_p and r_q, respectively. Γ_p and Γ_q are their Gini coefficients.*[16]

From portfolio theory, it is well known that the optimal market portfolio of all assets must be efficient in terms of risk-return tradeoff. That is, implicitly, all investors hold an optimal market portfolio by reallocating assets through longing (buying) and shorting (selling) activities in an aggregate sense. Let r_M be the market return such that $r_M = \sum_{i=1}^{n} \alpha_i r_i$, and the *cdf* of r_M be $F_m(r_M)$. Investors are implicitly maximizing the expected utility of r_M, $\underset{\alpha_i}{Max}\, EU(r_M)$, subject to $\sum_{i=1}^{n} \alpha_i = 1$. Thus, if investors view asset *i* as superior to asset *j*, then the long position of asset *i* increases (increase of α_i) and the short position of dominated asset *j* increases (decrease of α_j). Shalit and Yitzhaki (1994) show that in the portfolio optimization framework, assets *i* dominates *j* for all concave utility function if and only if

(5) $$\int_{-\infty}^{\tau_p^m} \int_{-\infty}^{\infty} r_i f(r_i, r_M) dr_M dr_i \geq \int_{-\infty}^{\tau_p^m} \int_{-\infty}^{\infty} r_j f(r_j, r_M) dr_M dr_j, \ \textit{for all p, where } 0 \leq p \leq 1,$$

[16] The Gini coefficient of r_p distribution can be written as Γ_p = 2Cov(p, $F(r_p)$), where $F(r_p)$ is the cumulative density function of r_p.

where $\tau_p^m = F_m^{-1}(p)$. Inequality (5) is the rule of Marginal Conditional Stochastic Dominance (MCSD). which has been simplified by Chow (2001).

To simply the MCSD rule, let $I_M^{\tau^m}$ be an indicator variable such that $I_M^{\tau^m} = 1$ if $r_M \leq \tau^m$, and $I_M^{\tau^m} = 0$, otherwise. Then,

(6) $$E(r_i I_M^{\tau^m}) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} r_i I_M^{\tau^m} f(r_i, r_M) dr_M dr_i = \int_{-\infty}^{\tau^m} \int_{-\infty}^{\infty} r_i f(r_i, r_M) dr_M dr_i ,$$

and the MCSD rule as shown in the inequality (5) can be written

(7) $$E(r_i I_M^{\tau^m}) \geq E(r_j I_M^{\tau^m}) \quad \text{for all } \tau^m .$$

$E(r_i I_M^{\tau^m})$ is the expected return of asset *i* for all corresponding market returns below the target τ^m generated from the market distribution. The difference between SSD and MCSD is that under the SSD rule shown in inequality (3), distributions of returns are ordered stand-along according to their own cumulative probability functions; but under the MCSD rule as it appears in (7), return distributions are ranked conditionally according to the distribution of the core or market portfolio.

To illustrate that SSD and MCSD may yield different result, let's consider the following numerical example. Suppose we have the following vectors of r_i, r_j, and r_M :

r_i	r_j	r_M
2	3	2
2	1	3
2	1	0
2	0	-1
1	0	-2
-4	0	-3

Both r_i and r_j have equal means, with the variance of r_j smaller than that of r_i. Note that the distributions of r_i and r_j are not symmetric and non-normal. The r_j distribution is positively skewed, and the r_i distribution is negatively skewed. In addition, r_i is relatively sensitive to the down-side of the market return, and r_j is positively correlated with up-side market movements.

Without any modeling specification, the distributions of r_i and r_j are ranked according the SSD rule as follows:

P	τ_p^i	SSD ordinate		τ_p^j	SSD ordinate
1/6	-4	-0.67	<	0	0.00
2/6	1	-0.50	<	0	0.00
3/6	2	-0.17	<	0	0.00
4/6	2	0.17	=	1	0.17
5/6	2	0.50	>	1	0.33
1	2	0.83	=	3	0.83

It appears that the SSD ordinates between asset *i* and asset *j* cross. This indicates that the distributions of r_i and r_j are non-comparable. The existence of no dominance suggests that asset *i* is as efficient as asset *j*. However, by employing the MCSD rule according to (7), We show that r_i is clearly dominated by r_j.

τ^m	Corresponding r_i	MCSD ordinate		Corresponding r_j	MCSD ordinate
-3	-4	-0.67	<	0	0.00
-2	1	-0.50	<	0	0.00
-1	2	-0.17	<	0	0.00
0	2	0.17	=	1	0.17
2	2	0.50	<	3	0.63
3	2	0.83	=	1	0.83

This demonstrates that the stand-along SSD ranking method ignores the sensitivity of assets returns to market conditions. MCSD is able to resolve this problem without losing the generality of the SSD approach. Therefore, MCSD is a powerful tool for ranking asset/portfolio performance ranking. Technically, the key difference between SSD and MCSD rules is that the SSD ranking employs different sets of targets, τ_p^i and τ_p^j from each individual distribution. However, the MCSD rule uses a common set of targets τ^m generated from the market return distribution.[17] In fact, this makes the statistical inference of the MCSD very straightforward.

To demonstrate the statistical inference procedure of MCSD, we begin by selecting a set of target returns, $\left\{\tau_t^m \middle| t = 1, 2, ..., m\right\}$, corresponding to a set of empirical quantiles of the market portfolio return distribution. Further, let

$$\Phi_{i-j}^{\tau_t^m} = E(r_i I_M^{\tau_t^m}) - E(r_j I_M^{\tau_t^m}) \tag{8}$$

There are three possible outcomes from the MCSD test: equality ($\Phi_{i-j}^{\tau_t^m} = 0$ for all t) ; dominance ($\Phi_{i-j}^{\tau_t^m} > 0$ for some *t*, but $\Phi_{i-j}^{\tau_t^m} = 0$ for the rest of *t*); and non-comparability ($\Phi_{i-j}^{\tau_t^m} > 0$ for at least one *t*, and $\Phi_{i-j}^{\tau_t^m} < 0$ for at least one *t*). Since conventional goodness of fit testing methods (e.g. *Chi-square* and *F*-test) are unable to distinguish between dominance and non-comparability when the null hypothesis of equality is rejected, a multiple comparison test becomes necessary. It is also important to note that, although using empirical quantiles from the market return sample as targets may involve sampling variation from the population quantiles, data snooping bias is limited. In fact, MCSD tests two portfolios' distributions conditionally on the same market return distribution. Therefore, the target selection procedure is independent of sampling distributions of the portfolio returns. Since the targets are intended to capture a set of finite points of return information from the market return distribution, how robust and/or consistent the

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[17] Since the targets are common for both asset *i* and asset *j*, the expression of the abscissa *p* in the notation of τ^m is not important and can be omitted.

sample quantile is to the population quantile, is unimportant to the nature of the MCSD test. Importantly, by employing the target approach, the statistical inference of MCSD is simple and straightforward.

Suppose portfolio returns follow a random walk process. Given a set of *N* random sample returns, $\{ (r_{i_1}, r_{j_1}, r_{M_1}), \ldots, (r_{i_N}, r_{j_N}, r_{M_N}) \}$, the sample estimates of MCSD ordinates can be expressed as:

$$\hat{\Phi}_{i-j}^{\tau_t^m} = N^{-1} \sum_{k=1}^{N} (r_{i_k} I_{M_k}^{\tau_t^m}) - (r_{q_k} I_{M_k}^{\tau_t^m}) . \qquad (9)$$

Chow (2001) shows that $\sqrt{N}(\hat{\Phi}_{i-j}^{\tau_t^m} - \Phi_{i-j}^{\tau_t^m})$ is asymptotically and normally distributed with a zero mean and a variance, $(\sigma_{i-j}^{\tau_t^m})^2$, such that

$$(\sigma_{i-j}^{\tau_t^m})^2 = (\sigma_i^{\tau_t^m})^2 + (\sigma_j^{\tau_t^m})^2 - 2Cov_{ij}^{\tau_t^m} \text{, where} \qquad (10)$$

$$(\sigma_i^{\tau_t^m})^2 = E[(r_i I_M^{\tau_t^m})^2] - [E(r_i I_M^{\tau_t^m})^2],$$

$$(\sigma_j^{\tau_t^m})^2 = E[(r_j I_M^{\tau_t^m})^2] - [E(r_j I_M^{\tau_t^m})^2], \text{ and}$$

$$Cov_{ij}^{\tau_t^m} = E[(r_i I_M^{\tau_t^m})(r_j I_M^{\tau_t^m})] - [E(r_i I_M^{\tau_t^m}) E(r_j I_M^{\tau_t^m})].$$

Thus, under the null hypothesis $H_o : \{\Phi_{i-j}^{\tau_t^m} = 0 | t = 1, ..., m\}$, the appropriate test statistic is

$$Z_{i-j}^{\tau_t^m} = \sqrt{N} \frac{\hat{\Phi}_{i-j}^{\tau_t^m}}{\hat{\sigma}_{i-j}^{\tau_t^m}}, \qquad (11)$$

for *t=1,…,m*, respectively, where $\hat{\sigma}_{i-j}^{\tau_t^m}$ is the estimated standard deviation.

Following Chow and Denning (1993), and letting the largest absolute value of the test statistic be $Z_{i-j}^{*\tau_t^m} = \underset{1 \le t \le m}{Max} \left| Z_{i-j}^{\tau_t^m} \right|$, the confidence interval for the extreme statistic can be defined as

$Z^{*\tau_t^m}_{i-j} \pm SMM(\alpha; m; \infty)$, where $SMM(\alpha; m; \infty)$ is the asymptotic critical value of the α point of the Studentised Maximum Modulus (*SMM*) distribution with parameter *m* and ∞ degrees of freedom. Thus, the asymptotic joint confidence interval of at least 100(1-α) percent for a set of MCSD estimates is:

(12) $$Z^{*\tau_t^m}_{i-j} \pm SMM(\alpha; m; \infty) \quad \text{for } t = 1,2,\ldots,m\ .$$

One can control the size of a multiple test of MCSD estimates by simply comparing the *Z*-statistics with *SMM* critical values. The empirical MCSD rules using the above inference procedure are summarized as follows:

Empirical MCSD Inference Rules*:*

(a) *Asset i dominates (is dominated by) asset j, if* $Z^{\tau_t^m}_{i-j} \geq (\leq)\ SMM(\alpha; m; \infty)$ *for all t and with at least one strong inequality.*

(b) *No dominance exists otherwise.*

Chow (2001) demonstrates that although the MCSD test is conservative in nature, it has power to detect dominance for samples with more than 300 observations, and is robust under both homoskedasticity and heteroskedasticity.

III. EMPIRICAL EXAMINATION

The empirical illustrations use data provided by Kenneth French for both the U.S. and international equity markets.[18] To examine the performance of *value* portfolios in the U.S. market, the MCSD approach is applied to monthly returns on the market and six value-weighted portfolios (B/L, B/M, B/H, S/L, S/M, S/H). Value portfolios are formed on size and book to market ratios from all NYSE, AMEX, and NASAQ stocks. All returns are from the period 1926 to 2002. In addition, sample data are decomposed into two subsets, one for the period 1926 to

1974, and another for the period 1975 to 2002. By creating a set of ten equally spaced percentiles, such that $Q_1 = 0.1$, $Q_2 = 0.2, \ldots, Q_{10} = 1.0$, a set of target returns is determined by the corresponding quantiles of the market return distribution. Target returns are reported for three cases: (1) the overall sample of 1926-2002, (2) the 1926-1974 sample, and (3) the 1975-2002 sample. Portfolio performance, or the dominance condition, is then examined by computing the test statistic of equation (8) for a set of MCSD ordinates and by further comparing the test statistic with the joint (*SMM*) critical value.

Under the MCSD framework, a portfolio of stocks outperforms the market if the conditional probability distribution of the stock portfolio ranks above the distribution of the underlying market portfolio. Statistically, this means that the MCSD test statistics comparing the ranking ordinates of the stock portfolio *p* with the market portfolio *M*, denoted as $Z_{p-M}^{\tau^m}$, should be non-negative with at least one statistic that is greater than the *SMM* critical value. Table 1 reports the MCSD tests for six value-weighted stock portfolios of different size and value (B/L, B/M, B/H, S/L, S/M, and S/H). To control for the test size, we compare the test statistics with the *SMM* critical value of 2.81 for the 5 percent level of significance.[19] It appears that the portfolios of small sized growth stocks (S/L) are dominated by the market portfolio, in that the $Z_{p-M}^{\tau^m}$ statistics for the overall sample (1926-2002) and the sub-samples (1926-1974 and 1975-2002) are mostly negative and are generally below the critical value of -2.81. Importantly, no significant positive statistics exist.

Interestingly, as shown in Table 1, there is a crossing MCSD ranking for the portfolios of S/M and S/H over the sample period 1926-2002. The MCSD Z-statistics of the S/M and S/H portfolios at the largest quantile (*Q*=1.0) are significantly positive (3.09 and 3.33, respectively),

[18] The data are available online at http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/.

but the Z-statistics for lower quantiles (Q ≤ 0.3) are significantly negative (e.g. –2.87 and -4.12 at Q = 0.3, respectively). Note that from equations (3) and (5), the MCSD ordinate, $\Phi_{p-M}^{\tau^m}$, at Q = 1.0 (or the largest target point) is the difference of mean returns between a stock portfolio and the market portfolio. The $\Phi_{p-M}^{\tau^m}$ ordinate at the Q = 0.3 quantile represents the (downside) conditional mean-difference of returns between the sample portfolio *p* and market portfolio *M* when the market return is less than –0.011. Therefore, for the overall S/M and S/H samples, the MCSD test shows that, although the mean return on small-sized value stocks is higher than that of the market portfolio, the downside conditional mean return for small-sized value stocks is significantly lower than the downside expected market return. Consequently, small value stocks do not outperform the market in the U.S.[20]

Importantly, the MCSD analysis in Table 1 shows that the value stock anomaly may represent a data-snooping bias and/or a sample-specific result. That is, there exists a conflict in dominance results between the pre- and post-1975 samples. For the sample period 1975-2002, value stocks (B/H) statistically dominate the market portfolio, and the market portfolio statistically dominates growth stocks (B/L). However, this out-performance (under-performance) of value (growth) stocks does not exist in the 1926-1974 sub-sample. In fact, value stocks (B/H) are dominated by the market portfolio, and growth stocks dominate the market portfolio in the pre-1975 sample. The conflicting MCSD orderings between pre-1975 and post-1975 also exist for the S/M and S/H portfolios. Small sized, high value stocks (S/H) dominate the market in the post-1975 period, but the market outperformed the small-sized value

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[19] The *SMM* critical values are available in Stoline and Ury (1979).

[20] Note that crossing, shown as the existence of both significantly positive and significantly negative statistics, implies non-comparability between two distributional orderings. The small-sized, high value stocks of S/H and S/M dominate the market for the period 1975-2001, but the market dominates S/H and S/M during 1926-1975. This may explain why the crossing MCSD ranking exists in the overall sample (1926-2001).

stocks before 1975. As a result, over the entire sample period (1926 to 1975), we observe a crossing (non-comparable) ranking (negative statistics for lower quantiles (e.g. Q<0.5) and positive statistics for higher quantiles (e.g. Q>0.5)) between small-sized, high value stock portfolios and the market portfolio. In addition, different from large firms, the dominance (significant z-statistics) of the post-1975 high value small (S/M and S/H) stocks appears upside of the market return distribution. In summary, Table 1 indicates that the value effect may exist after 1975. However, the effect of firm *size* appears to be more important than that of *value* before 1975. The post-1975 analysis implies that large cap value stocks provide downside risk protection, and small cap value stocks may have large upside potential.

To further examine the value and size effects, Table 2 presents the pair-wise cross-sectional MCSD comparisons among the 6-portfolios B/L, B/M, B/H, S/L, S/M, and S/H for the post-1975 samples. No dominance exists between the two value portfolios (B/H and S/H). However, B/H and S/H significantly dominate other portfolios. Particularly, B/H and S/H strongly dominate growth stocks (B/L and S/L). This is consistent with the previous analysis from Table 1. This indicates that the value effect is more significant than the size effect after 1975. Furthermore, if we define the efficient set of portfolios as the set such that no portfolio within the set is dominated by any other portfolio in the set, the MCSD ranking summary in Table 2 demonstrates that only the high value portfolios (B/H and S/H) are in the efficient set. This evidence suggests that the returns of the high book-to-market (B/M) value portfolios can serve as a factor, regardless of firm size, in determining the return-generating process of stocks.

The next empirical illustration uses the international stock market data of Kenneth French. The MCSD test is employed to compare the monthly return distributions of value versus growth portfolios conditionally on the overall or world market portfolio for January 1975 to

December 2001. For each country, the target returns are determined by the empirical quantiles of the world market return distribution. Table 3 reports the MCSD ranking results of value versus growth based on four different valuation criteria: book-to-market (B/M), earnings-to-price (E/P), cash earnings-to-price (CE/P), and dividend yield-to-price (D/P). Kenneth French forms the portfolios at the end of December each year by sorting one of the four ratios (B/M, E/P, CE/P, and D/P) and then computing value-weighted returns for the following 12 months. The value portfolios (High) contain firms in the top 30% and the growth portfolios (Low) contain firms in the bottom 30%. From Table 3.1, the MCSD test generally does not support the Fama and French (1998) empirical findings, in that only five of twenty-one countries, including Australia, Belgium, Japan, Spain, and the U.S., show that B/M value stocks statistically dominate B/M growth stocks.[21] By changing the valuation criterion from B/M to earnings-to-price (E/P), Table 3.2 shows that four countries, Australia, Japan, Hong Kong and the U.S., have consistent MCSD dominance of value stocks over growth stocks. However, the value effect of Belgium and Spain vanishes. Interestingly, it appears that the value effect may vary from country to country by different valuation criteria. Furthermore, if we restrict earnings to be cash earnings only, it appears, from Table 3.3, that in addition to Australia, Hong Kong, and the U.S., the CE/P has an effect in determining the out-performance of value stocks in Germany. Japanese equity market has no value effect on the CE/P criterion, although it is significantly sensitive to the E/P and B/M. Finally, the MCSD test is applied to the value-growth data using the dividend yield-to-price (D/P) criterion. From Table 3.4, one third of the 21 global markets, including Australia, France, Hong Kong, Japan, Malaysia, Switzerland, and the U.S., show significant dominance of value stocks (with high D/P) over growth stocks (with low D/P). In fact, the MCSD test

[21] Fama and French (1998) found that value stocks outperform growth stocks in twelve of thirteen international equity markets during the 1975-1995 period.

statistics appear to be much stronger than those in Tables 3.1, 3.2 and 3.3. This suggests that the D/P criterion is more effective in distinguishing value and growth stocks than any of the other criteria evaluated. In summary, the MCSD tests show that there is no growth-equity portfolio in all countries that outperform the market. Although the value effect exists for some countries, the effect varies by different valuation criteria.

To examine which valuation criterion is superior in determining value stocks, we test the dominance condition among all value portfolios according to four different criteria of B/M, E/P, CE/P, and D/P, respectively.[22] Table 4 presents the MCSD ranking results for the 14 international equity markets. It appears that the B/M criterion is superior to the D/P criterion in the German market, and the B/M value stocks dominate the E/P value stocks in the Japanese market. However, the Dividend Yield-to-Price seems to be a more effective criterion than the B/M method in the stock markets of Malaysia and Switzerland. For the remaining countries, there is actually no difference among the four valuation criteria in determining value stocks. This is because there is no statistical MCSD dominance for all possible rankings among value stock portfolios. Note that value stocks dominate growth stocks for all valuation methods shown in Table 3. From Table 4, the pair-wise MCSD rankings of US value stocks from different valuation criteria shows no dominance. This indicates that B/M, E/P, CE/P, and D/P are equally effective in discriminating value and growth stocks in the US equity market.

Finally, the MCSD is used to test the out-performance of the U.S. value stocks to other countries' value stocks. It appears that in Table 5, the value portfolios of Japan and Netherlands are dominated by these of the U.S. according all four criteria of B/M, E/P, CE/P and D/P. The U.S. equity market dominates the equity market portfolios of Italy, Japan, Netherlands, and

[22] Since from Table 3, value stocks are not generally dominated by the growth stocks, the analysis focuses on only value stocks.

United Kingdom, and no international equity market dominates the U.S. market. This indicates the U.S. market out-performed the world market during our sample period of Post-1975. It is important to note that neither the U.S. value stocks dominate the value stocks of Australia, Belgium, Singapore, Spain, Sweden, and Switzerland, nor any non-US value- portfolios dominates U.S. value stocks.

IV CONCLUSION

Beginning from Fama and French (1992, 1993), two fundamental firm attributes, the market equity (ME) and the ratio of book equity to market equity (B/M), have been well documented. Smaller stocks have higher average returns than larger stocks, and those firms with high B/M have higher average returns than firms with lower B/M. The tendency of value (small) stocks to outperform growth (large) stocks is not an anomaly. It can be viewed as risk factors, in equilibrium, priced in addition to the traditional CAPM-type systematic risk. Consequently, the value premium and size premium must be included in identifying the return generating process for equity.

This article argues that before specifying value and size to be additional factors in pricing financial assets, one must cautiously ensure the existence of out-performance of value and/or size portfolios. Without assumptions about the forms of investors' utility functions and that of return distributions, stochastic dominance (SD) is a powerful tool to examine the condition of dominance. Unfortunately, the traditional SD compares distributions independently without considering the joint nature between assets and the market core portfolio. The newly developed marginal and conditional stochastic dominance (MCSD) is able to overcome this problem in that it ranks assets based on their conditional distributions on market conditions. By applying the

MCSD test of Chow (2000) to French's data for both the U.S. and international equity markets, we show that there is some weak evidence of the out-performance for the value stocks in the U.S. equity markets from the post-1975 data. The value stocks do not dominate growth stocks for the pre-1975 period. This clearly indicates that value effect is not an anomaly but could be simply a risk-factor such that a risk premium exists between the value and growth.

Further, the MCSD test is employed to examine value effect in international equity markets. It appears that the equity markets of Australia Hong Kong and Japan are quite similar to the U.S. market in that value stocks outperform growth stocks. Nevertheless, the markets in Europe and elsewhere show no effect of value over growth. Importantly, negative MCSD ordinates and statistics for many countries exists indicating there are no positive premiums of growth over value. Consequently, the value factor in the international equity-pricing model must be used cautiously.

Table 1.

Marginal Stochastic Dominance of Value and Growth Portfolios in US

To test the out-performance of value stocks, we calculate MCSD statistics Z^{τ}_{p-M} **(value stocks over the market),** for the portfolios of B/L, B/M, B/H, S/L, S/M, and S/H corresponding to empirical quantiles of market return distribution, $\tau = \hat{F}_m^{-1}(p)$. Monthly value premiums and market returns are obtained directly from Kenneth French's data library at http://mba.tuck.dartmouth.edu/pages/faculty/ken.french. The corresponding MCSD ordinates are statistically different from zero at the 5 percent level when compared the Z-score with the *SMM* critical value of 2.81. The CMD ordinate at p = 1.0 is equivalent to the unconditional mean return.

Target Return $\tau = \hat{F}_m^{-1}(Q)$	Quantile Q = 0.1	Q = 0.2	Q = 0.3	Q = 0.4	Q = 0.5	Q = 0.6	Q = 0.7	Q = 0.8	Q = 0.9	Q = 1.0
1926-2001	-0.051	-0.026	-0.011	0.002	0.013	0.023	0.034	0.047	0.064	0.383
1926-1974	-0.056	-0.029	-0.013	0.001	0.011	0.209	0.032	0.047	0.061	0.383
1975-2001	-0.039	-0.021	-0.010	0.004	0.015	0.025	0.038	0.047	0.066	0.141
					Z^{τ}_{p-M}					
B/L										
1926-2001	1.41	0.96	1.38	0.10	-0.92	-1.13	-0.37	-0.15	-0.41	-0.81
1926-1974	2.13	2.36	3.03*	2.54	2.10	1.86	2.01	1.95	0.97	0.39
1975-2001	-2.48	-2.33	-2.73	-3.61*	-4.58*	-4.32*	-3.35*	-2.72	-1.58	-0.86
B/M										
1926-2001	1.85	2.97*	3.13*	3.74*	3.82*	3.41*	1.74	0.87	0.04	0.48
1926-1974	-0.10	1.22	1.45	1.98	1.73	1.32	0.50	-0.11	-0.58	0.30
1975-2001	3.72*	3.56*	3.63*	3.75*	3.95*	3.92*	2.27	1.32	0.35	0.35
B/H										
1926-2001	-1.67	-1.60	-2.43	-1.51	-0.66	0.14	-0.07	0.10	0.53	2.26
1926-1974	-3.36*	-3.95*	-4.90*	-4.01*	-3.34*	-2.61	-2.15	-1.65	-0.71	2.07
1975-2001	3.41*	4.06*	4.48*	4.94*	5.60*	3.63*	3.25*	2.87*	1.66	1.05
S/L										
1926-2001	-5.73*	-5.57*	-5.76*	-5.12*	-5.23*	-3.47*	-2.50	-2.46	-1.08	0.64
1926-1974	-3.82*	-5.01*	-5.32*	-5.58*	-4.72*	-3.73*	-2.62	-2.57	-1.39	0.59
1975-2001	-5.55*	-5.06*	-4.31*	-4.52*	-4.03*	-3.53*	-1.49	-2.07	-0.98	0.45
S/M										
1926-2001	-3.59*	-3.42*	-2.87*	-2.27	-0.90	0.08	0.89	0.87	2.11	3.09*
1926-1974	-3.40*	-4.77*	-5.21*	-4.79*	-3.35*	-2.59	-1.85	-1.72	-0.28	1.95
1975-2002	0.61	1.60	2.27	2.22	2.97	3.55*	3.84*	3.72*	3.56*	2.88*
S/H										
1926-2001	-4.47*	-4.27*	-4.12*	-3.21*	-1.55	-0.54	0.41	1.57	2.94*	3.33*
1926-1974	-4.39*	-5.49*	-6.11*	-5.42*	-3.99*	-3.25*	-2.33	-2.04	-0.35	2.31
1975-2001	0.92	1.83	2.51	2.78	4.02*	4.43*	4.56*	4.13*	3.58*	2.83*

Table 2
Cross-sectional MCSD Ranking of Value-Growth Portfolios

To test the value effect, we calculate MCSD statistics $Z^{\tau}_{p_i - p_j}$ **(portfolio P_i vs. portfolio P_j),** among B/L, B/M, B/H, S/L, S/M, and S/H corresponding to empirical quantiles of market return distribution, $\tau = \hat{F}_m^{-1}(p)$. Monthly value premiums and market returns are obtained directly from Kenneth French's data library at http://mba.tuck.dartmouth.edu/pages/faculty/ken.french. The corresponding MCSD ordinates are statistically different from zero at the 5 percent level when compared the Z-score with the *SMM* critical value of 2.81. The CMD ordinate at p = 1.0 is equivalent to the unconditional mean return.

	$Z^{\tau}_{p_i - p_j}$									
$\tau = \hat{F}_m^{-1}(q)$	*q*=0.1	*q*=0.2	*q*=0.3	*q*=0.4	*q*=0.5	*q*=0.6	*q*=0.7	*q*=0.8	*q*=0.9	*q*=1.0
B/L vs. B/M	-3.83*	-3.53*	-3.70*	-4.17*	-4.75*	-4.60*	-3.03*	-2.04	-0.89	-0.59
B/L vs. B/H	-3.52*	-4.01*	-4.44*	-5.09*	-5.91*	-5.64*	-3.73*	-3.13*	-1.80	-1.09
B/L vs. S/L	2.96*	3.32*	3.36*	3.31*	2.49	2.08	0.60	0.11	-0.33	-0.63
B/L vs. S/M	-1.16	-1.96	-2.61	-2.97*	-3.88*	-4.26*	-4.19*	-3.85*	-3.34*	-2.56
B/L vs. S/H	-1.34	-2.07	-2.72	-3.25*	-4.54*	-4.82*	-4.65*	-4.13*	-3.35*	-2.54
B/M vs. B/H	-1.57	-2.29	-2.69	-3.03	-3.51*	-3.09*	-2.22	-2.48	-1.83	-1.02
B/M vs. S/L	4.00*	4.46*	4.72*	4.95*	4.56*	4.18*	1.86	1.07	0.16	-0.23
B/M vs. S/M	1.50	0.50	-0.17	-0.21	-0.68	-1.18	-2.06	-2.42	-2.91	-2.28
B/M vs. S/H	0.94	0.01	-0.59	-0.92	-1.99	-2.32	-3.07*	-3.34*	-3.35*	-2.58
B/H vs. S/L	4.00*	4.71*	5.22*	5.64*	5.65*	5.12*	2.50	1.90	0.84	0.20
B/H vs. S/M	2.89*	2.78	1.72	1.95	1.87	0.99	-0.44	-0.60	-1.43	-1.53
B/H vs. S/H	2.26	1.82	1.35	1.32	0.45	-0.20	-1.54	-1.76	-2.17	-2.10
S/L vs. S/M	-4.21*	-5.68*	-6.88*	-7.49*	-7.74*	-7.57*	-4.63*	-3.74*	-2.76	-1.66
S/L vs. S/H	-4.21*	-5.42*	-6.51*	-7.14*	-7.87*	-7.63*	-4.86*	-4.03*	-2.90*	-1.84
S/M vs. S/H	-1.03	-1.09	-1.18	-1.83	-3.34	-2.76	-2.49	-2.26	-1.46	-1.09

MCSD Ranking Summary

A ">" means that the country listed in the left column dominates the country in the top row. A "<" means that the country listed in the top row dominates the country listed in the left column. An "X" means no dominance.

	B/M	B/H	S/L	S/M	S/H
B/L	<	<	>	<	<
B/M		<	>	<	<
B/H			>	>	X
S/L				<	<
S/M					<

Table 3.1

MCSD Tests for International Value-Growth Equity Portfolios: Book-to-Market (B/M)

The MCSD test statistics are denoted as Z_{v-g}^{τ} **(Value Portfolio vs. Growth Portfolio)** corresponding to empirical quantiles of the world market index return distribution, $\tau = \hat{F}_m^{-1}(p)$. Monthly value premiums and market returns are obtained directly from Kenneth French's data library at http://mba.tuck.dartmouth.edu/pages/faculty/ken.french. The corresponding MCSD ordinates are statistically different from zero at the 5 percent level when compared the Z-score with the *SMM* critical value of 2.81. The CMD ordinate at p = 1.0 is equivalent to the unconditional mean return.

	Z_{v-g}^{τ}									
$\tau = \hat{F}_m^{-1}(q)$	q=0.1	q=0.2	q=0.3	q=0.4	q=0.5	q=0.6	q=0.7	q=0.8	q=0.9	q=1.0
Austria	0.90	1.22	1.05	1.60	2.29	2.14	2.30	2.31	2.04	1.42
Australia	1.92	1.60	3.43*	3.21*	4.32*	4.46*	4.54*	3.89*	3.60*	2.28
Belgium	2.18	2.90	2.98*	3.16*	3.32*	2.77	2.05	1.52	1.69	1.72
Denmark	1.91	0.87	0.95	1.04	0.98	0.69	0.38	0.48	0.08	-0.72
Finland	2.47	2.76	2.54	2.49	2.20	2.60	2.15	1.20	0.23	-1.03
France	0.30	0.94	1.35	0.96	1.09	1.18	1.48	1.48	1.09	1.49
Germany	0.63	0.33	0.57	0.97	1.72	1.80	2.01	2.59	2.78	2.41
Hong Kong	-0.90	-1.34	-0.86	-0.98	-0.75	-0.76	-0.05	0.24	-0.04	1.23
Ireland	-0.79	0.31	-0.43	0.20	0.33	0.30	-0.26	-0.55	-0.57	-0.53
Italy	1.82	1.81	1.01	1.02	1.10	0.50	0.74	0.97	0.14	0.52
Japan	2.77	3.05*	3.32*	3.94*	3.40*	3.53*	3.55*	3.56*	3.91*	3.46*
Malaysia	-1.52	-0.40	-0.14	0.00	-0.23	-0.32	-0.26	0.28	0.10	1.11
Netherlands	-0.91	-1.38	-1.65	-0.93	-1.15	-1.11	-0.80	-0.77	-0.41	-0.75
New Zealand	-1.66	-0.28	-0.98	-1.04	-1.13	-0.68	-0.91	-1.42	-1.57	-0.98
Norway	-0.28	-0.48	-0.62	-0.79	-0.50	-0.61	-0.10	-0.15	-0.13	0.21
Singapore	-0.52	-1.17	-0.86	-1.41	-1.33	-1.11	-1.03	-0.52	0.27	2.31
Spain	2.49	2.82*	3.06*	1.74	2.63	2.25	1.96	1.55	1.21	0.56
Sweden	1.69	1.72	0.39	0.01	0.03	0.22	-0.12	0.48	1.05	0.10
Switzerland	0.90	1.22	1.19	0.76	1.46	1.49	1.26	0.39	0.49	0.30
UK	0.48	1.07	0.78	0.63	1.52	1.26	1.63	1.82	1.91	1.49
USA	3.52*	4.20*	4.94*	5.60*	6.51*	6.37*	5.24*	4.36*	3.03*	2.07

Table 3.2

MCSD Tests for International Value-Growth Equity Portfolios: Earning-Price (E/P)

The MCSD test statistics are denoted as Z_{v-g}^{τ} **(Value Portfolio vs. Growth Portfolio)** corresponding to empirical quantiles of the world market index return distribution, $\tau = \hat{F}_m^{-1}(p)$. Monthly value premiums and market returns are obtained directly from Kenneth French's data library at http://mba.tuck.dartmouth.edu/pages/faculty/ken.french. The corresponding MCSD ordinates are statistically different from zero at the 5 percent level when compared the Z-score with the *SMM* critical value of 2.81. The CMD ordinate at p = 1.0 is equivalent to the unconditional mean return.

	Z_{τ}^{v-g}									
$\tau = \hat{F}_m^{-1}(q)$	q=0.1	q=0.2	q=0.3	q=0.4	q=0.5	q=0.6	q=0.7	q=0.8	q=0.9	q=1.0
Austria	0.52	1.25	1.57	2.23	2.16	1.93	2.00	1.04	0.87	1.04
Australia	3.17*	3.25*	4.34*	3.90*	4.87*	5.16*	4.91*	3.97*	3.22*	1.56
Belgium	1.01	1.20	0.90	1.00	1.43	0.93	0.54	0.68	0.63	0.40
Denmark	1.21	1.73	1.76	0.95	0.61	0.45	0.47	0.07	-1.07	-1.68
Finland	2.30	2.70	2.28	2.17	1.77	2.08	1.63	0.66	-0.16	-1.31
France	-0.49	0.61	0.69	0.18	0.52	0.44	0.69	0.40	0.41	1.11
Germany	1.10	0.45	0.18	0.65	1.12	1.09	1.26	1.68	1.43	1.51
Hong Kong	1.92	2.27	2.68	3.27*	3.74*	3.55*	3.03*	2.87*	2.11	1.30
Ireland	-1.20	-0.30	-0.25	-0.54	0.51	0.86	0.21	-0.23	-1.09	-0.69
Italy	0.89	1.57	0.49	-0.15	-0.12	-0.55	-0.72	-0.65	-1.09	-0.75
Japan	2.74	3.30*	3.81*	3.97*	4.57*	4.36*	4.28*	3.82*	3.87*	2.74
Malaysia	0.09	1.60	1.56	2.26	2.13	2.05	1.58	1.75	1.51	1.53
Netherlands	-1.29	-0.36	-0.30	0.07	-0.04	-0.21	0.77	0.85	1.63	1.45
New Zealand	-0.79	-1.50	-1.98	-1.44	-1.58	-1.65	-1.68	-2.18	-2.13	-0.80
Norway	0.15	-0.32	0.33	0.49	1.38	1.39	1.90	1.33	1.23	1.19
Singapore	1.28	1.81	2.55	2.32	2.16	2.41	1.48	1.23	0.73	0.60
Spain	1.29	1.43	1.38	1.52	2.42	2.12	2.11	2.09	2.13	1.69
Sweden	1.63	1.28	0.59	0.39	0.62	0.74	0.22	0.63	0.36	-0.12
Switzerland	1.59	1.28	1.74	1.57	2.23	1.65	1.62	1.01	1.16	0.48
UK	-0.33	0.53	0.85	1.17	1.89	1.98	1.90	1.22	1.42	0.92
USA	3.15*	3.61*	4.24*	4.97*	5.62*	5.66*	4.88*	4.61*	3.22*	2.72

Table 3.3

MCSD Tests for International Value-Growth Equity Portfolios: Cash Earning-Price (CE/P)

The MCSD test statistics are denoted as Z_{v-g}^{τ} **(Value Portfolio vs. Growth Portfolio)** corresponding to empirical quantiles of the world market index return distribution, $\tau = \hat{F}_m^{-1}(p)$. Monthly value premiums and market returns are obtained directly from Kenneth French's data library at http://mba.tuck.dartmouth.edu/pages/faculty/ken.french. The corresponding MCSD ordinates are statistically different from zero at the 5 percent level when compared the Z-score with the *SMM* critical value of 2.81. The CMD ordinate at p = 1.0 is equivalent to the unconditional mean return.

	Z_{τ}^{v-g}									
$\tau = \hat{F}_m^{-1}(q)$	q=0.1	q=0.2	q=0.3	q=0.4	q=0.5	q=0.6	q=0.7	q=0.8	q=0.9	q=1.0
Austria	0.24	0.60	0.92	2.01	2.18	2.34	2.39	2.40	2.11	1.48
Australia	3.63*	3.80*	4.60*	4.15*	5.04*	5.19*	5.59*	5.30*	4.65*	3.46*
Belgium	1.76	2.09	2.15	2.19	2.45	2.13	1.60	1.80	1.86	1.71
Denmark	2.21	1.73	1.76	0.95	0.61	0.45	0.47	0.07	-1.07	-1.68
Finland	2.01	2.71	2.56	2.48	1.81	1.79	1.27	0.41	-0.46	-1.84
France	-0.60	0.31	0.14	-0.27	0.18	0.67	0.95	1.17	0.75	1.44
Germany	1.60	0.99	1.51	1.89	2.24	2.17	2.20	2.90*	2.87*	2.50
Hong Kong	1.82	1.91	2.39	2.88*	3.54*	3.51*	2.82*	2.31	1.26	0.77
Ireland	-0.45	0.82	1.04	1.17	1.64	1.82	1.52	1.02	-0.14	0.18
Italy	0.35	-0.12	-1.21	-0.87	0.31	-0.04	0.82	1.36	1.03	1.93
Japan	2.38	2.38	2.39	2.37	2.13	1.98	1.94	1.90	2.70	2.39
Malaysia	-0.04	1.48	1.63	2.64	2.48	2.57	2.23	2.21	2.17	2.39
Netherlands	-1.43	-1.22	-0.98	-0.12	-0.38	-0.67	-0.30	-0.24	0.26	0.03
New Zealand	-1.21	-1.87	-2.07	-1.74	-1.42	-0.99	-1.04	-1.48	-1.63	-0.30
Norway	-0.14	-0.24	-0.05	-0.20	0.80	1.71	1.97	1.53	1.81	1.97
Singapore	1.22	1.34	1.65	1.83	1.72	1.80	1.28	1.21	0.75	0.94
Spain	0.34	0.57	0.84	0.94	1.56	1.36	1.63	1.28	1.18	1.43
Sweden	1.37	0.88	-0.30	-0.26	-0.20	0.13	-0.28	0.36	0.85	0.37
Switzerland	1.21	1.12	1.37	1.20	1.72	1.22	1.08	0.71	0.85	-0.13
UK	-0.16	0.31	0.81	1.12	2.12	2.20	2.49	2.21	2.20	1.77
USA	3.81*	4.36*	5.12*	6.13*	6.61*	6.87*	5.52*	4.57*	2.99*	2.04

Table 3.4

MCSD Tests for International Value-Growth Equity Portfolios: Dividend Yield-Price (D/P)

The MCSD test statistics are denoted as Z_{v-g}^{τ} **(Value Portfolio vs. Growth Portfolio)** corresponding to empirical quantiles of the world market index return distribution, $\tau = \hat{F}_m^{-1}(p)$. Monthly value premiums and market returns are obtained directly from Kenneth French's data library at http://mba.tuck.dartmouth.edu/pages/faculty/ken.french. The corresponding MCSD ordinates are statistically different from zero at the 5 percent level when compared the Z-score with the *SMM* critical value of 2.81. The CMD ordinate at p = 1.0 is equivalent to the unconditional mean return.

	Z_{τ}^{v-g}									
$\tau = \hat{F}_m^{-1}(q)$	q=0.1	q=0.2	q=0.3	q=0.4	q=0.5	q=0.6	q=0.7	q=0.8	q=0.9	q=1.0
Austria	-0.37	0.18	0.04	1.33	1.62	1.75	2.09	2.49	2.02	2.20
Australia	3.40*	3.79*	4.35*	3.97*	4.69*	4.35*	4.16*	3.32*	2.46	0.93
Belgium	0.92	0.41	0.89	1.04	0.73	0.30	0.10	-0.29	-0.10	-0.14
Denmark	0.12	-1.45	-0.65	-0.52	-0.51	-0.31	-1.04	-0.84	-1.86	-0.99
Finland	2.29	2.79	2.45	2.76	2.48	2.60	1.98	1.20	0.41	-1.19
France	1.19	2.41	2.78	2.88*	2.90*	3.16*	2.98*	2.66	2.03	2.01
Germany	1.86	2.11	1.57	1.50	1.69	2.01	2.29	2.35	2.05	0.82
Hong Kong	3.50*	4.52*	5.28*	5.87*	5.31*	4.92*	4.04*	2.86*	1.23	-0.13
Ireland	0.01	1.26	1.12	1.08	1.32	1.38	1.15	0.87	0.32	0.03
Italy	0.28	1.16	1.20	1.42	2.41	1.95	1.90	2.03	1.50	1.21
Japan	3.16*	2.99*	2.48	2.64	1.95	1.81	1.57	1.17	1.82	1.71
Malaysia	1.64	3.20*	4.44*	5.03*	3.97*	4.16*	4.02*	4.06*	2.85	2.05
Netherlands	1.26	0.54	1.05	0.88	0.59	0.34	1.09	1.41	1.79	1.19
New Zealand	0.24	-1.27	-0.55	-0.01	-0.82	-1.45	-1.63	-1.29	-0.62	-0.23
Norway	2.15	0.84	1.91	1.54	2.04	2.07	2.46	1.82	1.42	1.24
Singapore	1.06	1.07	1.93	2.27	1.63	1.10	1.07	0.72	0.53	0.59
Spain	1.43	1.35	1.60	1.07	1.98	2.20	2.17	2.71	2.39	1.63
Sweden	1.44	2.00	0.84	0.60	0.90	0.93	0.35	0.84	1.25	0.53
Switzerland	1.83	2.14	2.87*	2.94*	3.12*	2.85*	2.35	1.83	2.10	1.31
UK	-0.45	1.06	1.47	1.50	1.71	1.31	1.44	1.01	0.98	0.42
USA	4.72*	5.87*	7.11*	7.73*	7.75*	7.42*	5.11*	3.85*	2.23	0.66

Table 4

MCSD Ranking of Value Portfolios among Different Selection Criteria

BMV, EPV, CEPV and DPV are the value portfolios using B/M, E/P, CEP and D/P criteria respectively.
A ">" means that the country listed in the left column dominates the country in the top row. A "<" means that the country listed in the top row dominates the country listed in the left column. An "X" means no dominance.

	EPV	CEPV	DPV		EPV	CEPV	DPV
Australia				Netherlands			
BMV	X	X	X	BMV	X	X	<
EPV		X	X	EPV		X	X
CEPV			X	CEPV			X
Belgium				Singapore			
BMV	X	X	X	BMV	X	X	X
EPV		X	X	EPV		X	X
CEPV			X	CEPV			X
France				Spain			
BMV	X	X	X	BMV	X	X	X
EPV		X	X	EPV		X	X
CEPV			X	CEPV			X
Germany				Sweden			
BMV	X	X	>	BMV	X	X	X
EPV		X	X	EPV		X	X
CEPV			X	CEPV			X
Hong Kong				Switzerland			
BMV	X	X	X	BMV	X	X	<
EPV		X	X	EPV		X	X
CEPV			X	CEPV			X
Italy				UK			
BMV	X	X	X	BMV	X	X	X
EPV		X	X	EPV		X	X
CEPV			X	CEPV			X
Japan				USA			
BMV	>	X	X	BMV	X	X	X
EPV		X	X	EPV		X	X
CEPV			X	CEPV			X

Table 5

MCSD Ranking between the U.S. and International Value Portfolios among Different Selection Criteria

BMV, EPV, CEPV and DPV are the value portfolios using B/M, E/P, CEP and D/P criteria respectively. MKT is denoted as the market portfolios. A ">" means that the country listed in the left column dominates the country in the top row. A "<" means that the country listed in the top row dominates the country listed in the left column. An "X" means no dominance.

	United States				
	BMV	EPV	CEPV	DPV	MKT
Australia					
BMV	X	X	X	X	X
EPV	X	X	X	X	X
CEPV	X	X	X	X	X
DPV	X	X	X	X	X
MKT	<	<	<	<	<
Belgium					
BMV	X	X	X	X	X
EPV	X	X	X	X	X
CEPV	X	X	X	X	X
DPV	X	X	X	X	X
MKT	X	X	X	X	X
France					
BMV	X	X	X	X	X
EPV	<	<	<	<	<
CEPV	<	X	X	X	<
DPV	X	X	X	<	<
MKT	<	<	<	<	<
Germany					
BMV	X	X	X	X	X
EPV	<	X	X	<	<
CEPV	X	X	X	X	X
DPV	<	<	<	<	<
MKT	<	<	<	<	<
Hong Kong					
BMV	X	X	X	X	X
EPV	X	X	X	X	X
CEPV	X	X	<	<	X
DPV	X	X	X	X	X
MKT	X	X	X	X	X
Italy					
BMV	<	X	X	<	<
EPV	<	<	<	<	<
CEPV	<	X	X	X	<
DPV	<	X	X	<	<
MKT	<	<	<	<	<
Japan					
BMV	<	<	<	<	<
EPV	<	<	<	<	<
CEPV	<	<	<	<	<
DPV	<	<	<	<	<
MKT	<	<	<	<	<

	United States				
	BMV	EPV	CEPV	DPV	MKT
Netherlands					
BMV	<	<	<	<	<
EPV	<	<	<	<	<
CEPV	<	<	<	<	<
DPV	<	<	<	<	<
MKT	<	<	<	<	<
Singapore					
BMV	X	X	X	X	X
EPV	X	X	X	X	X
CEPV	X	X	X	X	X
DPV	X	X	X	X	X
MKT	X	X	X	X	X
Spain					
BMV	X	X	X	X	X
EPV	X	X	X	X	X
CEPV	X	X	X	X	X
DPV	X	X	X	X	X
MKT	X	X	X	X	X
Sweden					
BMV	X	X	X	X	X
EPV	X	X	X	X	X
CEPV	X	X	X	X	X
DPV	X	X	X	X	X
MKT	X	X	X	X	X
Switzerland					
BMV	<	X	X	X	X
EPV	X	X	X	X	X
CEPV	X	X	X	X	X
DPV	X	X	X	X	X
MKT	<	X	X	X	X
UK					
BMV	<	X	<	<	<
EPV	<	<	<	<	<
CEPV	X	X	X	<	<
DPV	<	<	<	<	<
MKT	<	<	<	<	<

References

Brennan, Michael J., Tarun Chordia, and Avanidhar Subrahmanyam, 1998, Alternative factor specifications, security characteristics, and the cross-section of expected stock returns, *Journal of Financial Economics* 49, 345-373.

Cass, David and Joseph Stiglitz, 1970, The structure of investor preferences ans asset return, and separability in portfolio allocation, *Journal of Economic Theory* 2, 122-160

Chan, K. C., Nai-fu Chen and David Hsieh, 1985, An Exploratory Investigation of the Firm Size Effect, *Journal of Financial Economics* 14, 451-471.

Chan, Louis, J. Karceski and J. Lakonishok, 1998, The Risk and Return from Factors, *Journal of Financial and Quantitative Analysis*, 33, 159-188.

Chan, Louis K. C., Narasimhan Jagadeesh, and Josef Lakonishok, 1996, Momentum strategies, *Journal of Finance* 51, 1681-1713.

Chen, Nai-fu and Feng Zhang, 1998, Risk and Return of Value Stocks, *Journal of Business* 71, 501-535

Chopra, Navin, Josef Lakonishok, and Jay R. Ritter, 1992, Measuring abnormal performance: Do stocks overreact?, *Journal of Financial Economics* 31, 235-268

Chordia, Tarun, and Lakshmanan Shivakumar, 2002, Momentum, business cycle, and time-varying expected returns, *Journal of Finance* 57, 985-1019.

Chow, K. Victor, 2001, Marginal conditional stochastic dominance, statistical inference, and measuring portfolio performance, *Journal of Financial Research* 24, 289-307.

Chow, K. Victor and K. C. Denning, 1993, A simple multiple variance ratio test, *Journal of Econometrics* 58, 385-401.

DeBondt, Werner, and Richard Thaler, 1985, Does the stock market overreact?, *Journal of Finance* 40, 793-805.

DeBondt, Werner, and Richard Thaler, 1987, Further evidence of overreaction and stock market seasonality, *Journal of Finance* 42, 557-581.

Dichev, I. D., 1998, Is the risk of bankruptcy a systematic risk?, *Journal of Finance* 53, 1131-1147.

Dittmar, R. F., 2002, Nonlinear Pricing Kernels, Kurtosis Preference, and the Cross-Section of Equity Returns, *Journal of Finance* 57, 369-403.

Fama, Eugene F. and Kenneth R. French, 1992, The cross-section of expected stock returns, *Journal of Finance* 47, 427-465.

Fama, Eugene F. and Kenneth R. French, 1993, Common risk factors in the returns on stocks and bonds, *Journal of Financial Economics* 33, 3-56.

Fama, Eugene F. and Kenneth R. French, 1995, Size and book-to-market factors in earnings and returns, *Journal of Finance* 50, 131-155.

Fama, Eugene F. and Kenneth R. French, 1996, Multifactor explanations of asset pricing anomalies, *Journal of Finance* 51, 55-84.

Fama, Eugene F. and Kenneth R. French, 1998, Value versus growth: the international evidence, *Journal of Finance* 53, 1975-1999.

Grundy, Bruce and J. Spencer Martin, 2001, Understanding the nature and the risks and the sources of the rewards to momentum investing, *Review of Financial Studies* 14, 29-78.

Hahn, G.J., and R.W. Hendrickson, 1971, A table of percentage points of the distribution of the largest absolute value of k Student t variates and its applications, *Biometrika* 58, 323-332.

Hadar, J. and W. R. Russell, 1969, Rules for ordering uncertain prospects, *American Economic Review* 59, 25-34

Hanoch, G. and H. Levy, 1969, The efficiency analysis of choices involving risk, *Review of Economic Studies* 36, 335-346.

Harvey, Campbell and Akhtar Siddique, 2000, Time-varying Conditional Skewness and the Market Risk Premium, *Research in Banking and Finance* 1, 25-58.

Jegadeesh, Narasimhan, and Sheridan Titman, 1993, Returns to buying winners and selling losers: Implications for stock market efficiency, *Journal of Finance* 48, 65-91.

Jegadeesh, Narasimhan, and Sheridan Titman, 2001, Profitability of momentum strategies: an evaluation of alternative explanations, *Journal of Finance* 56, 699-720.

Johnson, Timothy C., 2002, Rational momentum effects, *Journal of Finance* 57, 585-608.

Knez, Peter J. and Mark J. Ready, 1997, On the robustness of size and book-to-market in cross-sectional regressions, Journal of Finance 52, 1355-1382.

Levy, H., 1992, Stochastic dominance and expected utility: survey and analysis, *Management Science*, 38, 555-593.

Lintner, John, 1965, The Valuation of Risky Assets and the selection of Risky portfolios in stock portfolios and capital budgets, *Review of Economics and Statistics* 47, 13-37.

Loughran, Tim, 1997, Book-to-Market Across Firm Size, Exchange, and Seasonality: Is There an Effect?, *Journal of Financial and Quantitative Analysis* 32, 249-268.

MacKinlay, A. Craig, 1995, Multifactor models do not explain deviations from the CAPM, *Journal of Financial Economics* 38, 3-28.

Richards, Anthony J., 1997, Winner-loser reversals in national stock market indices: Can they be explained?, *Journal of Finance* 52, 2129-2144.

Ross, Stephen, 1976, The arbitrage theory of capital asset pricing , *Journal of Economic* 13, 341-360

Ross, Stephen, 1978, Mutual fund separation and financial theory – the separating distribution, *Journal of Economic Theory* 17, 254-286.

Rothschild, M. and J. E. Stiglitz, 1970, Increasing risk I: a definition, *Journal of Economic Theory* 2, 225-243.

Rouwenhorst, K. Geert, 1998, International momentum strategies, *Journal of Finance* 53, 267-284.

Shalit, H. and S. Yitzhaki, 1994, Marginal conditional stochastic dominance, *Management Science* 40, 670-684.

Shalit, H. and S. Yitzhaki , 2003 An asset allocation puzzle: comment, *American Economic Review* 93, 1002-1008.

Sharpe, W.F., 1964, Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk, *Journal of Finance* 19, 425-442.

Stoline, M. R. and H. K. Ury, 1979, Tables of the studentized maximum modulus distribution and an application to multiple comparisons among means, *Technometrics* 21, 87-93,