行政院國家科學委員會專題研究計畫 成果報告

供應鏈中退化性產品在儲存空間有限且允許延遲付款下的最佳存貨訂購策略之研究(第 2 年)

研究成果報告(完整版)

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供應鏈中退化性產品在儲存空間有限且允許延遲付款下的

最佳存貨訂購策略之研究

Abstract：

This work presents an inventory model for optimizing the replenishment cycle time for a single deteriorating item under permissible delay in payments and constraints of warehouse capacity (own warehouse capacity, and so excess inventory is stored in the rental warehouse). Rented warehouses are assumed to charge higher unit holding costs than owned warehouses. Furthermore, item deterioration rates are assumed to differ between warehouses. This study has two main purposes：First, the mathematical models of the inventory system are established under the above condition. Second, this study demonstrates that the optimal solution not only exists but is unique and two theorems are devised for determining the optimal replenishment cycle time. Finally, numerical examples are presented to illustrate the above theorems.

Keywords： Inventory, EOQ, Two-warehouse system, Deterioration, permissible delay in payments

1. Introduction

During the past two decades, the influence of a permissible delay in payments on the optimal inventory system has attracted attention from numerous researchers. Goyal (1985) derived an EOQ model under conditions of permissible delay in payments. Subsequently, some research articles related to Goyal (1985) such as Aggarwal and Jaggi (1995) presented the economic ordering policies of deteriorating items in the presence of permissible payment delay. Chu et al. (1998) investigated the economic ordering policy of deteriorating items presented in the model of Aggarwal and Jaggi (1995). Furthermore, Jamal et al. (1997) generalized the model of Aggarwal and Jaggi (1995) to include consideration of shortages. Chen and Chung (1999) analyzed buyer economic order model under situations involving trade credit. Furthermore, Shah (1993 a) designed a inventory model for an exponentially decaying inventory under a situation of permissible payment delays. Finally, Hwang and Shinn (1997) designed a retailer pricing and lot-sizing policy for exponentially deteriorating products under the condition of permissible delay in payments. The other relevant papers related to the delay of payments such as Arcelus and Srinivasan (1993), Shah (1993 b), Chung (1998), Chung and Liao (2004, 2006), Daellenbach (1986), Haley and Higgins(1973), Jaggi and Aggarwal (1994), Jammal et al. (2000), Kim et al. (1995), Sarker et al. (2001), Shinn (1997), Shinn and Hwang (2003) and their references.

From an economic perspective, such credit policies can be applied as an alternative to price discounts to induce larger orders. Hence, a key problem associated with inventory maintenance is deciding where to stock the items. However, all the above models were developed primarily for a single warehouse with unlimited capacity. Additionally, thinking in more practical terms, any warehouse has limited capacity. Hence, the models listed above are unsuitable for situation where it is necessary to hold large stocks. An early discussion dealing with an inventory model with two storage facilities is presented by Hartely (1976) who designed an inventory model involving two different warehouses, one of which is owned (OW) while the other is rented (RW). The rented warehouse is used to store units in excess of the fixed capacity W of the owned warehouse, and it is assumed that the holding cost for the RW is exceeds that of the OW. Consequently, items are stored first in the OW, with only excess stock being stored in the RW. Sarma (1983) developed a deterministic inventory model with infinite production rate and two levels storage. Goswami and Chaudhuri (1992) devised two-storage models including those both with and without shortages, assuming linearly increasing time dependent demand. These models only discussed cases of non- deteriorating items were discussed. Sarma (1987) first developed a two-warehouse model for deteriorating items with infinite replenishment rate and shortages. Pakkala and Achary (1992a) further considered the two-warehouse model for deteriorating items with finite replenishment rate and shortages. Furthermore, Hiroaki and Nose (1996) discussed an inventory perishable inventory control with two customer types and different selling prices under the warehouse capacity constraint. Additionally, Benkherout (1997) relaxed the assumption of Sarma (1987) to be stocked in the OW. Yang (2004) developed the two-warehouse model for deteriorating items with shortages and inflation. Finally, Zhou and Yang (2005) established a two-warehouse model with stock-level-dependent demand rate and consideration of transportation cost. Many related articles can be found in Pakkala and Achary (1992b), Lee (2006), Yang (2006) and their references.

Based upon the above argument, this study develops an order-level inventory model for deteriorating items with two-storage facility and a permissible payment delay. This study assumes that suppliers are willing to provide retailers with a permissible delay in payments to stimulate product demand, and moreover assumes that item deterioration rates differ between the two warehouses. An expression for the total profit of the inventory system is derived for the proposed model. The optimal solution not only exists but also is unique. Finally, two theorems are designed to determine the optimal replenishment cycle time and numerical examples are presented to illustrate these theorems. These results can help retailers decision in making decision regarding whether or not to rent RW to hold more items to maximize annual profits.

2. Notations and Assumptions

The notations adopted in this paper are as below.

A the replenishment cost per order；

D the demand rate per unit time；

W the storage capacity of OW；

T the length of replenishment cycle；

Q the replenishment quantity per replenishment；

s the selling price per unit item；

c the purchasing cost per unit item；

h_0 the holding cost per unit per unit time in OW；

h_r the holding cost per unit per unit time in RW and $h_r \geq h_0$；

α the deterioration rate in OW, where $0 < \alpha < 1$；

β the deterioration rate in RW, where $0 < \beta < 1$ and $\beta > \alpha$；

T_w the time at which the inventory level reaches zero in RW；

T_a $\frac{1}{\alpha}\ln(1+\frac{\alpha W}{D})$；

M credit period set by the supplier ($M < T_a$)；

I_p capital opportunity cost (as a percentage)；

I_e earned interest rate (as a percentage)；

$\pi(T)$ the annual net profit function；

The following assumptions are used in developing the model.

(1) Replenishment is instantaneous with a known, constant lead time.

(2) The time horizon of the inventory system is infinite.

(3) Shortages are not allowed to occur.

(4) The owned warehouse (OW) has a fixed capacity of W units.

(5) The rented warehouse (RW) has unlimited capacity.

(6) The items of OW are consumed only after consuming the items kept in RW.

(7) The inventory costs (including holding cost and deterioration cost) in the RW are higher than those in the OW. That is, $h_r + \beta c > h_0 + \alpha c$.

(8) The supplier proposes a certain credit period and sales revenue generated during the credit period is deposited in an interest bearing account with rate I_e. At the end of this period, the credit is settled and the remaining balance will be paid off by loan form the bank with interest charges in stock with rate I_p if necessary.

(9) After the credit period, the retailer makes payment to the bank immediately after the sales of the items until the retailer pays off the remaining balance.

(10) $I_0(t)$ denotes the inventory level at time $t \in (0, T_w)$ in the OW in which the inventory level is decreasing only owing to deterioration of the item, $I_R(t)$ denotes the inventory level at time $t \in (0, T_w)$ in RW in which the inventory level is decreasing to zero due to demand and deterioration of the item. $I(t)$ denotes the inventory level at time $t \in (T_w, T)$ in which the inventory level is dropping to zero due to demand and deterioration of the item；

3. Mathematical formulation

The problem discussed in this investigation is how retailers know whether or not to rent RW to hold more items to maximize annual net profit function, $\pi(T)$, under above situation. To answer this question, this study first observes that if the order quantity $Q \le W$, then it is not necessary to rent the warehouse. Otherwise, W units of items are stored in the OW and the remainder are dispatched in the RW. Herein, if we denote $T_a = \ln(1+\alpha W / D)/\alpha$ (See Appendix A), the inequality $Q \le W$ holds if and only if $T_a \ge T$. As implied above, two situations can arise：(A) the single-warehouse inventory system and (B) the two-warehouse inventory system. This work focuses on the situation of two-warehouse inventory system. Moreover, the two-warehouse inventory system can be summarized as follows: At the beginning of each replenishment cycle, the system receives Q units of which W units are kept in the OW, while the remainder are kept in the RW. Besides, during the interval $(0,T_w)$, the items in the RW are consumed first, followed by those in the OW. Thinking in more practical terms, the deterioration depends on preserving facilities and environmental conditions available in a warehouse, different warehouses may have different deterioration rates. Accordingly, the inventory in the RW depletes because of deterioration and demand over time T_w until it reaches level zero. Thus, the variation of $I_R(t)$ at time $t \in (0,T_w)$ is given by the following differential equation：

$$\frac{dI_R(t)}{dt} + \beta I_R(t) = -D, \qquad 0 < t < T_w \tag{1}$$

with boundary condition $I_R(T_w) = 0$.

The solution of Eq. (1) is

$$I_R(t) = \frac{D}{\beta}\left[e^{\beta(T_w - t)} - 1\right], \qquad 0 \le t \le T_w \tag{2}$$

On the other hand, by this stage, a portion of the inventory in the OW is depleted owing to deterioration and consumption begins in the OW at time T_w. Thus, the variation of $I_0(t)$ at time $t \in (0,T_w)$ is determined by the following equation：

$$\frac{dI_0(t)}{dt} = -\alpha I_0(t), \qquad 0 < t < T_w \tag{3}$$

with initial condition $I_0(0) = W$. The solution to (3) is

$$I_0(t) = W \cdot e^{-\alpha t}, \qquad 0 \le t \le T_w \tag{4}$$

Finally, during (T_w,T), the inventory depletes as a result of the combination effect of demand and deterioration until time T. This behavior can be represented using the following differential equation：

$$\frac{dI(t)}{dt} + \alpha I(t) = -D, \quad T_w < t < T \tag{5}$$

with boundary condition $I(T) = 0$. The solution of (5) is

$$I(t)=\frac{D}{\alpha}\left[e^{\alpha(T-t)}-1\right] \quad , \quad T_w \le t \le T \tag{6}$$

Accordingly, the order quantity over the replenishment cycle is

$$Q=I_R(0)+I_0(0)=\frac{D}{\beta}(e^{\beta T_w}-1)+W \tag{7}$$

and the diagram of inventory level in this condition is shown in Fig. 1.

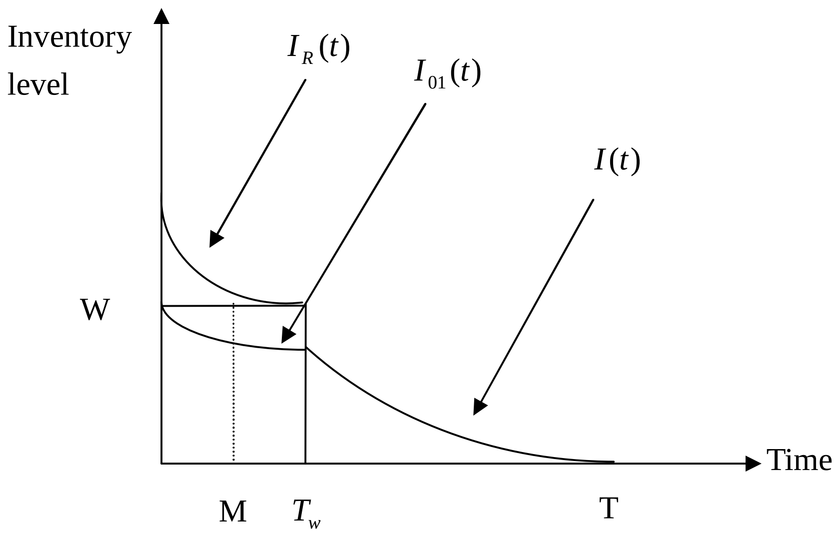

Fig. 1 $T_a < T$

Likewise, the annual net profit comprises the following elements.

(1) Annual sales profit= $D(s-c)$

(2) Annual ordering cost = $\frac{A}{T}$

(3) Annual stockholding cost in the RW$=\frac{h_r}{T}\int_0^{T_w} I_R(t)dt=\frac{h_r D}{\beta^2 T}(e^{\beta T_w}-\beta T_w-1)$

(4) Annual stock holding cost in the OW$=\frac{h_0}{T}[\int_0^{T_w} I_0(t)dt+\int_{T_w}^{T} I(t)dt]$

$$=\frac{h_0}{\alpha^2 T}[\alpha W(1-e^{-\alpha T_w})+D(e^{\alpha(T-T_w)}-\alpha(T-T_w)-1)]$$

(5) Annual purchasing cost $=\frac{cQ}{T}=\frac{c}{T}[\frac{D}{\beta}(e^{\beta T_w}-1)+W]$

(6) Annual capital opportunity cost:

Since $M<T_a<T$, the retailer sells the items and continues to accumulate sales revenue and earns the interest with rate I_e during time 0 to M. Hence, the interest earned per cycle is $DsI_eM^2/2T$ as well. In addition, the retailer starts paying the

interest for the items in stock after time M with rate I_p. Hence, the interest payable per cycle is

$$cI_p[\int_M^{T_w} I_R(t)\,dt + \int_M^{T_w} I_0(t)\,dt + \int_{T_w}^{T} I(t)\,dt]/T$$

$$= \frac{cI_p}{T}[\frac{D}{\beta^2}(e^{\beta(T_w-M)} - \beta(T_w - M) - 1) + \frac{W}{\alpha}(e^{-\alpha M} - e^{-\alpha T_w})$$

$$+ \frac{D}{\alpha^2}(e^{\alpha(T-T_w)} - \alpha(T - T_w) - 1)]$$

Therefore, annual capital opportunity cost=

$$\frac{cI_p}{T}[\frac{D}{\beta^2}(e^{\beta(T_w-M)} - \beta(T_w - M) - 1) + \frac{W}{\alpha}(e^{-\alpha M} - e^{-\alpha T_w}) + \frac{D}{\alpha^2}(e^{\alpha(T-T_w)} - \alpha(T - T_w) - 1)]$$

$$-\frac{DsI_e M^2}{2T}$$

Finally, annual net profit function is expressed as

$$\begin{aligned} \pi_3(T) = D(s-c) - \frac{A}{T} - \frac{h_r D}{\beta^2 T}(e^{\beta T_w} - \beta T_w - 1) - \frac{h_0}{\alpha^2 T}\{\alpha W(1 - e^{-\alpha T_w}) \\ + D[e^{\alpha(T-T_w)} - \alpha(T - T_w) - 1]\} - \frac{c}{T}[\frac{D}{\beta}(e^{\beta T_w} - 1) + W] \\ - \frac{cI_p}{T}\{\frac{D}{\beta^2}[e^{\beta(T_w-M)} - \beta(T_w - M) - 1] + \frac{W}{\alpha}(e^{-\alpha M} - e^{-\alpha T_w}) \\ + \frac{D}{\alpha^2}[e^{\alpha(T-T_w)} - \alpha(T - T_w) - 1]\} + \frac{DsI_e M^2}{2T} \end{aligned} \tag{8}$$

Notably, the value of $I_0(T_w)$ and $I(T_w)$ should coincide and thus

$$I_0(T_w) = W \cdot e^{-\alpha T_w} = \frac{D}{\alpha}[e^{\alpha(T-T_w)} - 1] = I(T_w) \tag{9}$$

Thereafter, substituting Eq. (9) into Eq.(8), this study rewrites the annual net profit function, $\pi_3(T)$, using the following segmented function：

$$\begin{aligned} \pi_3(T) = D(s-c) - \frac{A}{T} - \frac{h_r D}{\beta^2 T}(e^{\beta T_w} - \beta T_w - 1) - \frac{h_0}{\alpha T}[W - D(T - T_w)] - \frac{c}{T}[\frac{D}{\beta}(e^{\beta T_w} - 1) + W] \\ - \frac{cI_p}{T}\{[\frac{D}{\beta^2}(e^{\beta(T_w-M)} - \beta(T_w - M) - 1)] + \frac{1}{\alpha}[We^{-\alpha M} - D(T - T_w)]\} + \frac{DsI_e M^2}{2T} \end{aligned} \tag{10}$$

Additionally Eq. (9) implies that

$$T_w = \frac{1}{\alpha}\ln(\frac{De^{\alpha T} - \alpha W}{D}) \tag{11}$$

Notably, T_w is a function of T. Consequently, taking the first-order derivation of T_w

with respect to T, it yields

$$\frac{dT_w}{dT} = \frac{De^{\alpha T}}{De^{\alpha T} - \alpha W} > 1 \tag{12}$$

Based on profit consideration, the necessary condition for $\pi_3(T)$ to be maximized is that the first derivative of $\pi_3(T)$ is zero and thus this study obtains

$$\begin{aligned}\pi_3'(T) = \frac{1}{T^2}\{A &- \frac{h_r D}{\beta^2}(\beta T e^{\beta T_w}\frac{dT_w}{dT} - \beta T\frac{dT_w}{dT} - e^{\beta T_w} + \beta T_w + 1)\\ &- \frac{h_0}{\alpha}(DT\frac{dT_w}{dT} - DT_w - W) - c(DTe^{\beta T_w}\frac{dT_w}{dT} - W - \frac{D}{\beta}e^{\beta T_w} + \frac{D}{\beta})\\ &- cI_p[\frac{D}{\beta}T(e^{\beta(T_w - M)} - 1)\frac{dT_w}{dT} - \frac{D}{\alpha}T(1 - \frac{dT_w}{dT}) - \frac{D}{\beta^2}(e^{\beta(T_w - M)} - \beta(T_w - M) - 1)\\ &- \frac{1}{\alpha}(We^{-\alpha M} - D(T - T_w)] - \frac{DsI_e M^2}{2}\}\end{aligned} \tag{13}$$

Furthermore, $\pi_3'(T) = 0$ hold if and only if the right side of Eq. (13) equals zero.

Let

$$\begin{aligned}f_3(T) = A &- \frac{h_r D}{\beta^2}(\beta T e^{\beta T_w}\frac{dT_w}{dT} - \beta T\frac{dT_w}{dT} - e^{\beta T_w} + \beta T_w + 1)\\ &- \frac{h_0}{\alpha}(DT\frac{dT_w}{dT} - DT_w - W) - c(DTe^{\beta T_w}\frac{dT_w}{dT} - W - \frac{D}{\beta}e^{\beta T_w} + \frac{D}{\beta})\\ &- cI_p \cdot h(T) - \frac{DsI_e M^2}{2}\end{aligned} \tag{14}$$

where

$$\begin{aligned}h(T) = \frac{D}{\beta}T(e^{\beta(T_w - M)} - 1)\frac{dT_w}{dT} &- \frac{D}{\alpha}T(1 - \frac{dT_w}{dT}) - \frac{D}{\beta^2}(e^{\beta(T_w - M)} - \beta(T_w - M) - 1)\\ &- \frac{1}{\alpha}(We^{-\alpha M} - D(T - T_w))\end{aligned}$$

where both $f_3(T)$ and $\pi_3(T)$ share the same sign and domain. The derivative of $f_3(T)$ with respect to T is

$$f_3'(T) = -DT\{(\frac{h_r + \beta c}{\beta})[\beta(\frac{dT_w}{dT})^2 + (\frac{d^2 T_w}{dT^2})]e^{\beta T_w} - (\frac{\alpha h_r - \beta h_0}{\alpha\beta})(\frac{d^2 T_w}{dT^2})\} - cI_p \cdot h'(T) \tag{15}$$

where

$$h'(T) = \frac{DT}{\beta}\{\beta(\frac{dT_w}{dT})^2 + \frac{d^2 T_w}{dT^2} + (\frac{\beta - \alpha}{\alpha})(\frac{d^2 T_w}{dT^2})\cdot e^{-\beta(T_w - M)}\}\cdot e^{\beta(T_w - M)}$$

Since

$$\beta(\frac{dT_w}{dT})^2 + (\frac{d^2T_w}{dT^2}) > \alpha(\frac{dT_w}{dT})^2 + (\frac{d^2T_w}{dT^2}) = \frac{\alpha De^{\alpha T}}{(De^{\alpha T} - \alpha W)} > 0$$

and

$$\frac{d^2T_w}{dT^2} = -\frac{\alpha^2 DWe^{\alpha T}}{(De^{\alpha T} - \alpha W)^2} < 0$$

so, we have

$$\begin{aligned} h'(T) &> \frac{DT}{\beta}\{\beta(\frac{dT_w}{dT})^2 + \frac{d^2T_w}{dT^2} + (\frac{\beta-\alpha}{\alpha})(\frac{d^2T_w}{dT^2})\cdot 1\}\cdot e^{\beta(T_w - M)} \\ &= \frac{DT}{\alpha}\{\alpha\,(\frac{dT_w}{dT})^2 + (\frac{d^2T_w}{dT^2})\ \}\cdot e^{\beta(T_w-M)} \\ &> 0 \end{aligned}$$

Finally, for the sake of completeness, let us consider the following two cases：

(a) If $\alpha h_r \ge \beta h_0$, then $f_3'(T) < 0$. That is, $f_3(T)$ decreases on $(0,\infty)$.

(b) If $\alpha h_r < \beta h_0$, then it is impossible to ensure that $f_3'(T)$ is less below 0.

Combining (a) and (b) attention in the remainder of the mathematical analysis is limited to the conditions of $\alpha h_r \ge \beta h_0$.

Let T_3^* denote the roots of $f_3(T) = 0$ if T_3^* exist. Then, the following theorem is obtained.

Theorem 1：

(1)If $f_3(T_a) > 0$, T_3^* is the unique solution such that $\pi_3(T)$ has the maximum on $[T_a,\infty)$.

(2)If $f_3(T_a) \le 0$, then $\pi_3(T)$ is decreasing on $[T_a,\infty)$. So $T_3^* = T_a$.

Proof：

(1)If Eq. (13) is rewritten, then

$$\pi_3'(T) = \frac{1}{T^2} f_3(T) \tag{16}$$

Since $f_3(T)$ is a strictly decreasing function of T. From $\lim_{T\to\infty} f_3(T) = -\infty$ and $f_3(T_a) > 0$, the Intermediate Value Theorem (1996)[30] yields that $f_3(T) = 0$ has a unique solution T_3^* on $[T_a,\infty)$. Hence

$$f_3(T)\begin{cases} >0 & if \quad T \in (T_a, T_3^*) \\ =0 & if \quad T = T_3^* \\ <0 & if \quad T \in (T_3^*, \infty) \end{cases}$$

which implies

$$\frac{d\pi_3(T)}{dT}\begin{cases} >0 & if \quad T \in (T_a, T_3^*) & (17a) \\ =0 & if \quad T = T_3^* & (17b) \\ <0 & if \quad T \in (T_3^*, \infty) & (17c) \end{cases}$$

Therefore, $\pi_3(T)$ is increasing on $[T_a, T_3^*]$ and decreasing on $[T_3^*, \infty)$. Restated, $\pi_3(T)$ is unimodal and the optimal value of $\pi_3(T)$ is T_3^*.

(2)If $f_3(T_a) \le 0$, then $f_3(T) < 0$ for all $T \ge T_a$ implying $\pi_3'(T) < 0$ for all $T > T_a$. That is, $\pi_3(T)$ is decreasing on $[T_a, \infty)$. So $T_3^* = T_a$.

We have completed the proof.

4. The optimization with two-warehouse model

From the perspective of a practical situation, retailers will not want to know the optimal order policy of the single-warehouse system or the two-warehouse system with payment delay, respectively. Retailers wish to know whether to use the rented warehouse to store significantly more items when the supplier offers a credit period. Because holding inventory exceeding W units by using RW in this situation may bring more profits to the retailer. As described above, the annual net profit function of the proposed model is derived as follows:

$$\pi(T) = \begin{cases} \pi_1(T) & if \quad 0 < T \le M & (18a) \\ \pi_2(T) & if \quad M < T \le T_a & (18b) \\ \pi_3(T) & if \quad T_a < T & (18c) \end{cases}$$

where

$$\pi_1(T) = D(s-c) - \frac{A}{T} - \frac{h_0 D}{\alpha^2 T}(e^{\alpha T} - \alpha T - 1) - \frac{cD}{\alpha T}(e^{\alpha T} - 1) + DsI_e(M - \frac{T}{2}) \qquad (19)$$

$$\pi_2(T) = D(s-c) - \frac{A}{T} - \frac{h_0 D}{\alpha^2 T}(e^{\alpha T} - \alpha T - 1) - \frac{cD}{\alpha T}(e^{\alpha T} - 1) - \frac{cI_p D}{\alpha^2 T}(e^{\alpha(T-M)} - \alpha(T-M) - 1) + \frac{DsI_e M^2}{2T} \tag{20}$$

and

$$\begin{aligned}\pi_3(T) = {} & D(s-c) - \frac{A}{T} - \frac{h_r D}{\beta^2 T}(e^{\beta T_w} - \beta T_w - 1) - \frac{h_0}{\alpha T}[W - D(T - T_w)] \\ & - \frac{c}{T}[\frac{D}{\beta}(e^{\beta T_w} - 1) + W] - \frac{cI_p}{T}\{\frac{D}{\beta^2}(e^{\beta(T_w - M)} - \beta(T_w - M) - 1) \\ & + \frac{1}{\alpha}(We^{-\alpha M} - D(T - T_w))\} + \frac{DsI_e M^2}{2T}\end{aligned} \tag{21}$$

Furthermore, at $T = T_a$, given $T_w = 0$ and $W = \frac{D}{\alpha}(e^{\alpha T_a} - 1)$. From the following lemma, it can be determined that $\pi_2(T_a) < \pi_3(T_a)$.

Lemma 1: $\frac{1}{\alpha^2}(\alpha T - 1 + e^{-\alpha T}) - \frac{1}{\beta^2}(e^{-\beta T} - 1 + \beta T) > 0$ for $T > 0$.

Proof. The proof of Lemma 1 is shown in Appendix B.

Furthermore, Eqs. (20) and (21) yield

$$\pi_2(T_a) - \pi_3(T_a) = -\frac{cI_p D}{T_a}\{\frac{1}{\alpha^2}(\alpha M - 1 + e^{-\alpha M}) - \frac{1}{\beta^2}(e^{-\beta M} - 1 + \beta M)\}$$

Lemma 1 implies that $\pi_2(T_a) < \pi_3(T_a)$. Therefore, $\pi(T)$ is continuous except $T = T_a$.

Eqs. (19), (20) and (21) yield that

$$\pi_1'(M) = \pi_2'(M) = \frac{1}{M^2}[A - \frac{D(h_0 + \alpha c)}{\alpha^2}(\alpha M e^{\alpha M} - e^{\alpha M} + 1) - \frac{DsI_e M^2}{2}] \tag{22}$$

$$\begin{aligned}\pi_2'(T_a) = {} & \frac{1}{T_a^2}[A - \frac{D(h_0 + \alpha c)}{\alpha^2}(\alpha T_a e^{\alpha T_a} - e^{\alpha T_a} + 1) \\ & - \frac{cI_p D}{\alpha^2}(\alpha T_a e^{\alpha(T_a - M)} - e^{\alpha(T_a - M)} - \alpha M + 1) - \frac{DsI_e M^2}{2}]\end{aligned} \tag{23}$$

and

$$\pi_3'(T_a) = \frac{1}{T_a^2} \cdot f_3(T_a) \tag{24}$$

For convenience, this study lets

$$f_2(M) = A - \frac{D(h_0 + \alpha c)}{\alpha^2}(\alpha M e^{\alpha M} - e^{\alpha M} + 1) - \frac{DsI_e M^2}{2}$$

and

$$f_2(T_a) = A - \frac{D(h_0 + \alpha c)}{\alpha^2}(\alpha T_a e^{\alpha T_a} - e^{\alpha T_a} + 1)$$
$$- \frac{cI_p D}{\alpha^2}(\alpha T_a e^{\alpha(T_a - M)} - e^{\alpha(T_a - M)} - \alpha M + 1) - \frac{DsI_e M^2}{2}$$

Since $\pi_2(T)$ is concave on $[M,\infty)$, then $\pi_2'(T)$ is decreasing on $[M,\infty)$ and $f_2(M) > f_2(T_a)$. In addition, this study has $f_2(T_a) < f_3(T_a)$ and

$$f_2(M) > 0 \text{ if and only of } T_1^* > M \tag{25}$$

$$f_2(M) > 0 \text{ if and only of } T_2^* > M \tag{26}$$

$$f_2(T_a) > 0 \text{ if and only of } T_2^* > T_a \tag{27}$$

$$f_3(T_a) > 0 \text{ if and only of } T_3^* > T_a \tag{28}$$

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(*)此研究已被 Computers & Industrial Engineering 接受

無研發成果推廣資料

97 年度專題研究計畫研究成果彙整表

<table>
<tr><td colspan="3">計畫主持人：廖瑞容</td><td colspan="5">計畫編號：97-2221-E-263-001-MY2</td></tr>
<tr><td colspan="8">計畫名稱：供應鏈中退化性產品在儲存空間有限且允許延遲付款下的最佳存貨訂購策略之研究</td></tr>
<tr><td colspan="3" rowspan="2">成果項目</td><td colspan="3">量化</td><td rowspan="2">單位</td><td rowspan="2">備註（質化說明：如數個計畫共同成果、成果列為該期刊之封面故事...等）</td></tr>
<tr><td>實際已達成數（被接受或已發表）</td><td>預期總達成數(含實際已達成數)</td><td>本計畫實際貢獻百分比</td></tr>
<tr><td rowspan="12">國內</td><td rowspan="4">論文著作</td><td>期刊論文</td><td>0</td><td>0</td><td>100%</td><td rowspan="3">篇</td><td></td></tr>
<tr><td>研究報告/技術報告</td><td>0</td><td>0</td><td>100%</td><td></td></tr>
<tr><td>研討會論文</td><td>0</td><td>0</td><td>100%</td><td></td></tr>
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<tr><td>已獲得件數</td><td>0</td><td>0</td><td>100%</td><td></td></tr>
<tr><td rowspan="2">技術移轉</td><td>件數</td><td>0</td><td>0</td><td>100%</td><td>件</td><td></td></tr>
<tr><td>權利金</td><td>0</td><td>0</td><td>100%</td><td>千元</td><td></td></tr>
<tr><td rowspan="4">參與計畫人力（本國籍）</td><td>碩士生</td><td>0</td><td>0</td><td>100%</td><td rowspan="4">人次</td><td></td></tr>
<tr><td>博士生</td><td>0</td><td>0</td><td>100%</td><td></td></tr>
<tr><td>博士後研究員</td><td>0</td><td>0</td><td>100%</td><td></td></tr>
<tr><td>專任助理</td><td>0</td><td>0</td><td>100%</td><td></td></tr>
<tr><td rowspan="12">國外</td><td rowspan="4">論文著作</td><td>期刊論文</td><td>1</td><td>2</td><td>95%</td><td rowspan="3">篇</td><td></td></tr>
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<tr><td>研討會論文</td><td>0</td><td>0</td><td>100%</td><td></td></tr>
<tr><td>專書</td><td>0</td><td>1</td><td>100%</td><td>章/本</td><td></td></tr>
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<tr><td rowspan="2">技術移轉</td><td>件數</td><td>0</td><td>0</td><td>100%</td><td>件</td><td></td></tr>
<tr><td>權利金</td><td>0</td><td>0</td><td>100%</td><td>千元</td><td></td></tr>
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<tr><td>博士生</td><td>0</td><td>0</td><td>100%</td><td></td></tr>
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<tr><td>專任助理</td><td>0</td><td>0</td><td>100%</td><td></td></tr>
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其他成果 (無法以量化表達之成果如辦理學術活動、獲得獎項、重要國際合作、研究成果國際影響力及其他協助產業技術發展之具體效益事項等，請以文字敘述填列。)	食品 GMP 認證體系推展成效調查研究

	成果項目	量化	名稱或內容性質簡述
科教處計畫加填項目	測驗工具(含質性與量性)	0	
	課程/模組	0	
	電腦及網路系統或工具	0	
	教材	0	
	舉辦之活動/競賽	0	
	研討會/工作坊	0	
	電子報、網站	0	
	計畫成果推廣之參與（閱聽）人數	0	

國科會補助專題研究計畫成果報告自評表

請就研究內容與原計畫相符程度、達成預期目標情況、研究成果之學術或應用價值（簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性）、是否適合在學術期刊發表或申請專利、主要發現或其他有關價值等，作一綜合評估。

1. 請就研究內容與原計畫相符程度、達成預期目標情況作一綜合評估

   ■達成目標

   □未達成目標（請說明，以 100 字為限）

   　　□實驗失敗

   　　□因故實驗中斷

   　　□其他原因

   說明：

2. 研究成果在學術期刊發表或申請專利等情形：

   論文：■已發表 □未發表之文稿 □撰寫中 □無

   專利：□已獲得 □申請中 ■無

   技轉：□已技轉 □洽談中 ■無

   其他：（以 100 字為限）

3. 請依學術成就、技術創新、社會影響等方面，評估研究成果之學術或應用價值（簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性）（以 500 字為限）